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**Effects of Demand Managements on Sector Sizes
and Okun's Law**

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Abstract

This paper¹ examines the effects of attempts at altering the magnitudes of macroeconomic output, GDP, by changing the pattern of demand shares of sectors which comprise the economy. We have earlier proved² that larger shares of demands on a more productive subset of sectors results in a higher value of GDP. When demand patterns are changed before the model state reaches some neighborhood of an equilibrium, however, such pattern changes may reduce the magnitudes of GDP rather than increasing them. We explain this seemingly counter-intuitive behavior in this paper. We also examine the effects of endogenized shares of demands on the coefficients of the Okun's law, and provide conditions under which higher GDP values produce larger coefficient values.

Introduction

In Aoki (2002, Chapter 8) we have discussed a simple model of output fluctuations of an economy composed of K sectors. In this model sector i receives an exogenously chosen positive fraction s_i , $\sum_i s_i = 1$, of the total GDP, $Y = \sum_{j=1}^K Y_j$, where sector j 's contribution to GDP is denoted by Y_j . Sector i employs n_i factors of production (labor) with $Y_i = c_i n_i$, $i = 1, \dots, K$. In this paper productivity coefficients c_i 's are held fixed. Outputs of all sectors are assumed to be linear in the size of the sectors (number of employees or number of production lines and such).

The supply of the sector i goods is thus given by

$$Y_i = c_i n_i,$$

¹A preliminary version of this paper was presented at the LABORatorio Riccardo Revelli WILD@Ace 2004 meeting in December 2004.

²For a related model see models of outputs and fluctuations of Aoki (2002, Chapt. 8), and Aoki and Yoshikawa (2003).

and demand of sector i goods is $s_i Y$, with $Y = \sum_i c_i n_i$, where c_i is productivity coefficient, n_i is the size of sector i , and where s_i is a positive demand share of GDP, $\sum_i s_i = 1$.

The model has an continuum of equilibria defined by the vanishing of expected values of excess demands of all the sectors, i.e.,

$$E(f_i) = s_i E(Y^e) - c_i E(n_i^e) = 0,$$

for all i , where f_i is the excess demand of sector i , $E(\cdot)$ denotes expectation, and superscript e denote equilibrium values.

From these two relations we obtain the expression for the equilibrium size of sector i as

$$n_i^e = \frac{s_i}{c_i} Y^e. \quad (1)$$

By summing (1) over all sectors we have a relation between the equilibrium level of output and the size of the economy (the total number of employees)

$$E(L^e) = \kappa E(Y^e), \quad (2)$$

where

$$\kappa = \sum_{j=1}^K \frac{s_j}{c_j}.$$

Taking the ratio of (1) and (2), we note that the expected value of the equilibrium size of sector i is expressed as a fraction of the mean total size of the economy as

$$\frac{E(n_i^e)}{E(L^e)} = \frac{s_i/c_i}{\kappa}.$$

For simpler notation we suppress superscript e from now on.

In the next section we examine the effects of demand pattern, summarized by the coefficients $s_i, i = 1, 2, \dots, K$, on this relative size expression. Effects of changing the shares have some unexpected effects on sizes of sectors and on the magnitude of GDP as shown in the next section.

We then examine the effects of demand pattern on the coefficients of Okun's law. In this section we endogenize the demand share, and denote it as $s_i(Y), i = 1, 2, \dots, K$.

Stochastic Processes of Size Changes

By changing the demand patterns and starting the model from the same initial conditions, we observe how much time elapses before the model enters the closed set, and how large is the value of the stationary GDP. We observe from the simulations to be summarized at the end of this paper that the larger is the share on more productive sectors the faster are the model dynamics, and the earlier is the entry into stationary closed sets. This indicates that the dynamics are indeed nonlinear. We later describe the variations in the outputs and sector sizes (sectoral employments) in business cycles near one of the equilibria.

Fig. 1 show four GDP levels with four demand patterns of this model with $K=10$. The highest level is the GDP with 12/17 of the total demand is shared by the top 5 most efficient sectors. The bottom is the case where the top 5 most efficient sectors share only 5/17 of total GDP. With the scale used in the Fig. 1 the equilibrium levels look constant. These figures are the average of 400 Monte Carlo runs. Actually there are oscillations of small amplitude. Fig. 2 is one such example of GDP with 2/3 of the total GDP is shared by the top 5 most efficient sectors.

Unless all excess demands are zero, the model is not in equilibrium and some sector sizes change stochastically. At any given point in time, there are subsets of sectors with positive excess demands and of sectors with negative excess demands. Since all sectors are interconnected, as soon as one sector changes its size, GDP changes, hence the demands for all sectors also change as well as the subsets of positive and negative excess demands.

We assume that all sectors with positive excess demands wish to increase their sizes, by hiring or activating one additional unit of production factor, and that all sectors with negative excess demands wish to reduce their sizes either by firing an employee or halting one of the assembly lines, for example. Only one sector with non-zero excess demand can succeed in carrying out their plans for size changes. That sector is the one with the shortest holding (sojourn) time. See Aoki (2002, p.27–29, Sec. 8.6), or Lawler (1995) on holding times. In effect, the probability that a sector with positive or negative excess demand becomes active is given by the sizes of these subsets when no growth is involved. See Aoki for detail.

Moving from One Basin of Equilibrium to Another

Suppose that the demand pattern $s_i, i = 1, \dots, K$ is selected and the model moves from a fixed initial condition and settles into a basin containing the equilibrium corresponding to these s values. With another set of demand share pattern, the model, starting from the same initial sizes of the sectors, will generally settle into a different basin with different values of Y and N since the values of κ will be different. Demand policy is effective in the sense that more demand is placed on sectors with higher productivity coefficients, the higher becomes GDP. In Aoki (2002, p.109) we have established this fact analytically for $K = 2$.

Now suppose that before the model outputs approach near some equilibrium value, some of the s_i s are exogenously changed. This may be interpreted as the policy maker changing demand shares during some transition periods.

We next show two examples to illustrate a possibility that shifting more demand to more productive sector does not always yield higher GDP, if the switches of demand patterns occur **before** the model state reaches equilibria.

The reason has to do with the relative sizes of sectors. With high demands placed on goods produced by less efficient sectors, the equilibrium sizes of these sectors become larger to meet the demands than more efficient sectors. When more fractions of demands are shifted to more productive sectors, the less productive sectors must contract their size (labor), and the

more productive sectors must expand their sizes. The probability of contraction, however, is much higher than that of expansion, at least initially. Recall that only active sectors can change their sizes and the probability that a sector becomes active is proportional to the relative sizes of sectors. This leads to initial decline in GDP, and in some circumstances net loss of GDP results.

Two Examples

The next two simple examples illustrate how sector size changes determine the ratio of the equilibrium values of GDP before and after demand pattern shifts. To simplify explanations suppose that there are only two sectors, one more technically advanced than the other. Without loss of generality assume that $c_1 = 1$ and $c_2 = c < 1$.

The restriction of K to 2 may be interpreted as follows: Consider an economy with 10 sectors with a demand pattern $s_1 = \dots = s_5 = 1/15$, and $s_6 = \dots = s_{10} = 2/15$ and another demand pattern in which the first 5 sectors have demand share $2/15$ each, and the last 5 sectors $2/15$. Aggregating the shares of the first 5 and the last 5, this example corresponds to the one discussed above with $\sigma = 1/3$.³ Then the ratio of the sector size 2 before and after is such that n'_2/n_2 is less than $1/2$, where prime indicates the value after the switch.

In phase 1, we assume that demand pattern is $s_1 = \sigma$ and $s_2 = 1 - \sigma$, and in phase 2 demand pattern switches to $s'_1 = 1 - \sigma$ and $s'_2 = \sigma$. We set $\sigma = 1/3$ in both examples, that is, demand is higher for the less efficient sector 2 goods in phase 1, and for the more efficient sector 1 goods in phase 2.

In phase 1, the ratio of the two sectors are

$$\frac{n_1}{n_2} = \frac{\sigma}{1 - \sigma} c,$$

and in phase 2

$$\frac{n'_1}{n'_2} = \frac{1 - \sigma}{\sigma} c.$$

The outputs are

$$Y = n_1 + cn_2 = \frac{c}{1 - \sigma} n_2,$$

and

$$Y' = n'_1 + cn'_2 = \frac{c}{\sigma} n'_2.$$

Consequently, the ratio of outputs is

$$\frac{Y'}{Y} = \frac{1 - \sigma}{\sigma} \frac{n'_2}{n_2}.$$

The GDP value in phase 2 is less than that in phase 1 if and only if

$$\frac{n'_2}{n_2} < \frac{\sigma}{1 - \sigma}.$$

³This can be done in more general settings as well.

This inequality is satisfied for σ less than 0.5.

Example 1 The value of productivity coefficient is $c = 1/2$.

Set the value of $n_1 = m$, where m is some large positive number in phase 1. Then $n_2 = 4m$. In this model the inefficient sector 2 is 4 times larger than the efficient sector. In phase 2 we have $n'_1 = 2m$, and $n'_2 = 2m$, and $Y = Y' = 3m$.

In this example the size of sector 1 increases by m , that of sector 2 shrinks by $2m$, while the equilibrium values of the GDP becomes the same.

Example 2 The value of productivity coefficient is $c = 1/3$.

Let $n_1 = m$. Then $n_2 = 6m$, and $Y = 3m$. In phase 2 $n'_1 = 4m/3$, $n'_2 = 2m$, and $Y' = 2m$. In this example the output in phase 2 is less than phase 1.

When a higher demand is placed on the technically inefficient sector, the size of the sector 2, the less efficient one, becomes larger than that of sector 1. Suppose sectors reach the equilibrium. This is phase 1 equilibrium. Then, the demand pattern changes. Now the demand on the less efficient sector is reduced, and that on the more efficient one increases. This is phase 2. These two examples above show that under certain condition, the effects of size reduction of the inefficient sector outweighs the effects of size increases in the efficient sector, i.e., sector 1. The net changes in GDP consequently is a reduction of GDP.

Simple probability analysis

Here we outline the simple case. In the two sector case discussed above, we define the death rate by $\mu_n = n_2/n$ and the birth rate by $\lambda_n = n_1/n$, where $n = n_1 + n_2$. Initially we have $\mu_n > \lambda_n$ by design. The birth rate is the probability that an active sector is chosen from sector 2, causing the size of sector 2 to shrink by one unit. When sector 1 becomes an active sector its size grows by one unit.

Since the binary tree is huge in general, we can keep track of the sectors that are active in transition from phase 1 to phase 2, for example. We also can calculate the maximum likelihood estimate of number of times sector 2 becomes active from its initial size to a smaller size. This simple calculation shows that initially majority of active sectors come from sector 2.

Since the transition probabilities change as the sizes of the two sectors change, we merely indicate the relative probabilities of active sectors coming from sector 1 in a couple of time instants as snap shots. We take m large enough so that we ignore the changes in probabilities over a period of 20.

Start from the initial state $(n_1, n_2) = (m, 4m)$. During this period let us suppose that sector 1 becomes active k times and sector 2 l times, where $k + l = 20$. In other words, sizes change to $(m + k, 4m - l)$. Calculate the maximum likelihood of l . Then we see that l is most likely to be 16. In other words, sector 2 decreases initially in size with probability approximately 0.8.

As the size adjustment progresses, suppose the sizes change to $(n_1, n_2) = (m, 3m)$. The values of most likely l is now 15, with probability of decrease of size of sector 2 is 0.75. When the model reaches the sizes $(n'_1, n'_2) = (2m, 2m)$, the sizes have reached an equilibrium values, and the probability

of being active becomes 0.5 for both sectors.

We can intuitively understand this when we realize that the reductions in size of sector 2 occurs with higher probability than the probability of size increase of sector 1, because with higher probability the active sector is chosen from sector 2, at least initially.

More generally, the process of size changes caused by demand pattern shifts can be calculated by means of binary trees birth-and -death of units in the sectors. This can be represented as a binary tree. The left branch moves are associated with increases in n_1 , and rightward moves are associated with decreases in n_2 .

For example, if the binary tree starts from the initial state $A = (m, 4m)$ and reaches the equilibrium state $D = (2m, 2m)$, then the segment of the tree enclosed by $A = (m, 4m), B = (m, 2m), C = (2m, 4m), D = (2m, 2m)$ is the relevant section. Fig. 3 shows this, where increases in sector 1 only bounds the northwest side of the tetrahedron, and the northeast side represents decreases in sector 2 size while keeping the size of sector 1 intact. The southwest side represents decreases in only sector 2 size, and southeast side corresponds to increase in sector 1 size only. The interior of the tetrahedron contain (n_1, n_2) with both increases in sector 1 and decrease in sector 2.

Okun's Law

We define the Okun's law by

$$\frac{\Delta Y}{Y_e} = -\beta \left\{ \frac{\Delta U}{N} \right\}, \quad (3)$$

where $N = L + U$ is the total population of which L is employed and U is unemployed. In this paper we keep N fixed for simpler presentation.

This numerical value of β is much larger than what one expects under the the standard neoclassical framework. Take, for example, the Cobb-Douglas production function with no technical progress factor. Then, GDP is given by $Y = K^{1-\alpha}L^\alpha$ with α of about 0.7.

We have $\Delta U = -\Delta L$, where ΔK and ΔN are assumed to be negligible in the short run. The production function implies then that $\Delta Y/Y = \alpha \Delta L/L$ in the short run. That is, one percent decrease in Y corresponds to an increase of $\Delta U/N = -(1/\alpha)(\Delta Y/Y)(1 - U/N)$, i.e., an increase of a little over 1 percent of unemployment rate. To obtain the number 4, as in the Okun's law, we need some other effects, such as increasing marginal product of labor or some other nonlinear effects. See Yoshikawa (2000).

We assume that economies fluctuate about its equilibrium state, and refer to the relation (3) as Okun's law, where Y_e is the equilibrium level of GDP, approximated by the central value of the variations in Y in simulation. Similarly, ΔU is the amplitude of the business cycle oscillation in the unemployed labor force. U_e is approximated by the central value of the oscillations in U , and Y_e and L_e are related by the equilibrium relation (2).

The changes $\Delta Y/Y$ and $\Delta U/U$ are read off from the scatter diagrams in simulation after allowing for sufficient number of time to ensure that the model is in "stationary " state.

In simulations described below we note that after a sufficient number of time steps have elapsed, the model is in or near the equilibrium distribution. Then, Y and U are nearly linearly related with a negative slope, which can be read off from scatter diagrams i.e.,

$$\Delta Y = -x\Delta U.$$

From the equilibrium relation between $L = \kappa Y$ we note that

$$\Delta L = -\kappa\Delta Y.$$

By combining these two relations we obtain

$$\kappa x = 1,$$

and

$$\beta = 1 + \frac{U}{L},$$

which is much smaller than 4 per cent figure.

We next see that the situation changes as the shares s_i s are made to depend on Y .

Differentiate the continuum of equilibrium relation $L = \kappa Y$ (dropping superscript e from now on) with respect to Y to obtain

$$\Delta L = \kappa\left[1 + \frac{1}{\kappa}\frac{d\kappa}{dY}\right].$$

$$n_i = \kappa Y,$$

with

$$\kappa(Y) = \sum_i \frac{s_i(Y)}{c_i},$$

so that

$$\frac{d\kappa}{dY} = \sum_i \frac{1}{Y}\Theta,$$

with

$$\Theta = \sum_i \frac{s_i(Y)}{c_i} \frac{d \ln s_i(Y)}{d \ln y}.$$

Then the coefficient of the Okun's law becomes

$$\beta = -\left(1 + \frac{u}{L}\right)\left(1 + \frac{1}{\kappa}\frac{d\kappa}{dY}\right). \quad (4)$$

Okun's law in the economic literature usually refers to changes in gross domestic products (GDP) and unemployment rates measured at two different time instants, such as one year apart. There may therefore be growth or decline in the economies.

To avoid confusing the issues about the relations between GDP and unemployment rates during stationary business cycle fluctuations, that is, without growth of GDP, and those with growth, we run our simulations in

stationary states assuming no change in the numbers of sectors, productivity coefficients, or the total numbers of labor force in the model.

The Okun's law refers to a stable empirical relation between unemployment rates and rate of changes in GDP: one percent increase (decrease) in GDP corresponds to β percent decrease (increase) in unemployment, where β is about 4 in the United States.

Example

Let the shares vary according to

$$s_i(Y) = s_{0i} - \gamma_i(Y - Y^e),$$

$i = 1, 2$, where s_{0i} is the equilibrium value $s_i(Y^e)$, and $\gamma_1 + \gamma_2 = 0$ to satisfy the condition that the sum equals 1. Here are some numbers. With $(s_1 s_2) = (1 - s, s)$ where $s_{0i} = 0.1$, $i = 1, 2$, $(1/c - 1) = 10^2$, $\gamma_1 = 10^{-1}$ we obtain $\beta = 2.1$. With $\gamma = 1.5 \times 10^{-2}$, $\beta = 4.3$. With $s_0 = 4 \times 10^{-2}$, $1/c - 1 = 10^2$, $\gamma = 3 \times 10^{-2}$, $\beta = 3.6$.

Simulation studies

Since the model is nonlinear and possibly possesses multiple equilibria, we use simulations to deduce some of the properties of the models. We pay attention to the phenomena of trade-offs between GDP and unemployment, and the scatter diagrams of GDP vs. unemployment to gather information on business cycle behaviors.

Our model behaves randomly because the jumping sectors are random due to holding times being randomly distributed. This is different from the models in the literature which behave randomly by the technology shocks which are exogenously imposed. As we indicate later the state space of the model have many basins of attractions each with near equal output levels.

Simulations are used to gather information on model behavior.⁴

We report on two patterns of demands: $P_1 := [2, 2, 2, 2, 2, 1, 1, 1, 1, 1]/15$, and $P_2 := [1, 1, 1, 1, 1, 2, 2, 2, 2, 2]/15$.

Four hundred Monte Carlo runs of duration 7000 elementary time steps each have been run. Fig. 2 is the average GDP of P_1 . It shows that after 700 time steps the model is in the closed set. The lower panel shows the details of oscillations.

From Fig. 1 we see that, $E(Y^e) = 363$ with P_1 , and with $P_2 E(Y^e) = 230$. The average GDP is smaller with P_2 than with P_1 . The average standard deviation of Y during the time period [2000, 7000] is about 9.4 and 9.2 for the two patterns. The values of κ is 2.38 and 4.86 for P_1 and P_2 respectively.

With P_2 it takes about 1500 time steps for the model to enter the closed set, that is, the model is much more sluggish with P_2 than P_1 . Fig. 4 and 5 show the effects of switches from P_1 to P_2 and from P_2 to P_1 which takes

⁴Simulation programs were written originally by V. Kalesnik, a graduate student at UCLA, and later modified by, F. Ikeda, and M. Suda, graduate students at Univ. Tokyo.

place at $t=3000$. Timmings of switches also matter as shown in Fig. 6 and 7. In Fig. 6 the switch from P_1 to P_2 occurs at $t=500$, and at $t=1500$, while Fig.4 at $t=3000$.

Conclusions

We have demonstrated by simulation that higher percentages of demands falling on more productive sectors produce four new results: (1): average GDPs are higher; (2): the Okun's coefficients are larger; (3): transient responses are faster, and (4): timing of the demand pattern switches matter in changing GDP. Unlike the Cobb-Douglas production or linear production function which lead to x values of less than one, we obtain the values from 2 to 4 depending on κ as the Okun's coefficient in our simulation.

It is remarkable that we can deduce these results from models with linear constant coefficient production functions. This indicates the importance of stochastic interactions among sectors introduced through the device of stochastic holding times. Using a related model Aoki and Yoshikawa (2003) allow the positive excess demand sectors to go into overtime until they can fill the vacancy. This model produces the Beveridge curve shifts. The details are to appear in Aoki and Yoshikawa (2005).

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Fig. 1 Four examples of average values of GDP with four different demand share vectors

Fig. 2 Average GDP sample value for $P_1 = [2, 2, 2, 2, 2, 1, 1, 1, 1, 1]/15$ demand share vector.

Fig. 3 The segment of the binary tree to go from $(n_1, n_2) = (m, 4m)$ to an equilibrium state $(2m, 2m)$, where m is some large positive integer. The left-downward arrow indicates the direction of increase of n_1 . The right-downward arrow is the direction of n_2 decrease.

Fig. 4 The profile of the GDP average with switch from P_1 to P_2 at $t=3000$. The lower panel is an enlarged section near the switch time point.

Fig. 5 The enlarged profile of the GDP averages with switch from P_2 to P_1 at $t = 3000$. Note that Y^e is lower after the switch. Fig. 6 The average profile of GDP with switch from P_1 to P_2 taking place at $t = 500$. Note that Y^e is lower. Fig 7. same as Fig. 6 with the switch taking place at $t = 1500$. Note that the Y^e is about the same.