Modular Pyramidal Hierarchies and Social Norms. An Agent Based Model.

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Abstract

We provide a model of hierarchical organization where artificial agents with limited individual capacities allocate their efforts in two activities. Different incentives schemes are considered and individual diversity and social norms are approached.

Introduction

The Moral Hazard literature approaches multi-agent relationships in different ways. Among them, the *joint production models* provide interesting insights in terms of income distribution among the agents. Another relevant aspect has been the comparison between centralized and decentralized structures as far contracting goes. For example, the literature provides conditions under which the situation where all the contracts are proposed by the principal (centralized organization) is superior to a more decentralized one.

Following the joint production approach we consider a modular model of hierarchical organization. Specifically, we consider pyramidal structures. This particular structure is spread wise and, consequently, both the economic (see Beckmann, 1988, for a formal analysis) and simulative literature (for instance see Glance and Huberman, 1994) finds interest. For an analysis of the different approaches to pyramidal structures see Merlone (2003).

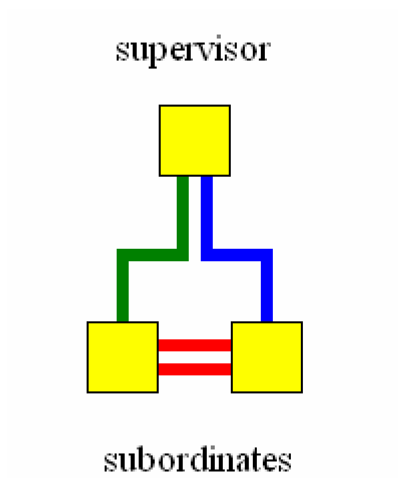
In our model, the organization consists of heterogeneous agents interacting in supervised teams with a Cobb-Douglas production function. We provide a theoretical analysis of the agents interaction in the modular element of the organization. Furthermore, we study the impact of heterogeneity of agents, social norms and incentive schemes in the organization. While in each team there exist infinite solutions to the optimal effort allocation problem, the presence of a social norm allows the selection of one of them.

When considering this simple model of supervised team embedded in a multi-layer organization, the complex dynamics between the agents call for the simulation approach. The Agent-Based simulation model we consider allows the extension of the results obtained in the theoretical analysis of the supervised team. First, using the simulation platform we develop, it is possible to consider complex dynamics where agents adapt their efforts to different incentive schemes and to the observable variables. Second, it is possible to study the overall performance of organization when considering different characteristics of the agents, namely managerial capacity, individual capacity, sensitivity to different social norms and rewards. Finally, different adaptive managerial behaviors are compared.

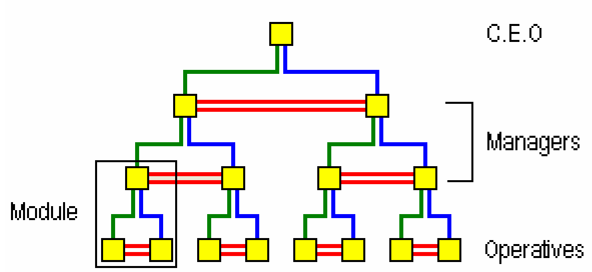
Our results shed light on some aspects of interaction between individuals in complex environment and economic performance and give insights in terms of observation of the performance measures in the organization. Furthermore, we prove that, while in general rewards based only on observable efforts by supervisor lead to the underperformance of the organization, under certain circumstances, this kind of rewards allows to an improvement in the performance when a social norm is considered.

The model

We consider a hierarchical modular model of organization. Each module consists of a supervisor and two subordinates as shown in the following figure.

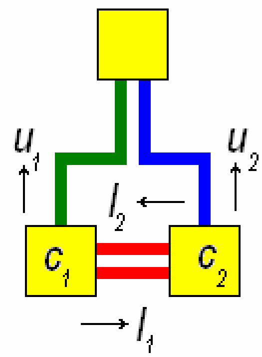


The organization we consider is modular i.e., each supervisor is paired with a same level one and both are in turn supervised by another manager. The organization may consist up to six layers of agents for a total of sixty-three agents. Obviously, the module is the most elementary organization, with only two layers. In the following figure we can see an example of a four layers organization where modules are well recognizable.



In the organization, bottom levels individuals are referred as operatives, middle level individuals are referred as managers and the top level individual is referred as the C.E.O.

As a consequence, in our hierarchy operatives play only the subordinate's role, the C.E.O. plays only the supervisor's role, while managers play both, i.e. they are at the same time supervisors and subordinates.
Let us consider a general organization where agents are identified by an index i belonging to the set {0, 1, …, 62}. Agent i has a capacity c_i to be allocated between effort l_i provided with its partner and effort u_i provided with its supervisor. The joint production function for agents 1 and 2 is $(u_1 + u_2)^\alpha (l_1 + l_2)^\beta$ where $0 < \alpha < 1$ and $0 < \beta < 1$ are respectively the output elasticity with respect to the joint effort with the supervisor and with the partner. The following picture illustrates the allocation of the efforts exerted by the two considered agents.



We assume that each agent's capacity is private information. For bottom level agents it remains constant over time while, for the others, it depends on the subordinates output. This is consistent to Quian (1994) where the final output of the hierarchy is determined by a production function which is cumulative in the efforts of workers and managers at all levels. Furthermore, each agent can observe the level effort his/her partners provides but cannot observe the effort his/her partner provides with the boss. By converse, the supervisor can observe the joint output and the effort each agent provides with her. Finally, we assume that each agent knows the average efforts provided with partners and supervisors in his/her level.
The agent's retribution is given by a fixed wage plus a linear incentive proportional to the joint output of the team and a linear incentive on the effort each agent exerts with the supervisor. The net output of the supervised team is the supervisor's capacity if she is up to work in some other supervised team, or the organization output if she is at the top of the organization.
Since, as we said above, a different place in the organization implies a different role, in the following we restrict our analysis to a single module considering the agents' and the supervisor's problem.

The agents' problem

We start considering agents in the subordinate role case, i.e., agents that stay at the very bottom of the hierarchy. The agents' problem is simple when no social norm is considered. Assuming coordination and commitment between agents, the following problem must be solved

$$\max_{u_1,u_2,l_1,l_2} \quad (u_1+u_2)^{\alpha}(l_1+l_2)^{\beta}$$
$$\text{sub} \qquad u_i + l_i \le c_i, \quad i = 1,2$$

By first order conditions it is easy to obtain

$$\begin{cases} u_1+u_2 = \dfrac{\alpha}{\alpha+\beta}(c_1+c_2) \\ l_1+l_2 = \dfrac{\beta}{\alpha+\beta}(c_1+c_2) \end{cases}.$$

There are infinite solution to the considered problem, nevertheless, a rather natural effort allocation is

$$\begin{cases} u_1 = \dfrac{\alpha}{\alpha+\beta}c_1 \\ l_1 = \dfrac{\beta}{\alpha+\beta}c_1 \end{cases} \qquad \begin{cases} u_2 = \dfrac{\alpha}{\alpha+\beta}c_2 \\ l_2 = \dfrac{\beta}{\alpha+\beta}c_2 \end{cases}.$$

This allocation may be interpreted as focal in the sense of Schelling (1960).
The process the agents use to reach this effort allocation is the following:

$$\begin{cases} \left(u_1^{t+1}, l_1^{t+1}\right) = BR\left(u_2^t, l_2^t\right) \\ \left(u_2^{t+1}, l_2^{t+1}\right) = BR\left(u_1^t, l_1^t\right) \end{cases}.$$

With any initial condition such that

$$\left(u_1^0, l_1^0\right) = \left(k_1\frac{\alpha}{\alpha+\beta}c_1, k_1\frac{\alpha}{\alpha+\beta}c_1\right), \quad \left(u_2^0, l_2^0\right) = \left(k_2\frac{\alpha}{\alpha+\beta}c_2, k_2\frac{\alpha}{\alpha+\beta}c_2\right) \quad k_1, k_2 \in [0,1],$$

the system converges to the natural effort allocation in one single step. From any other initial conditions the system oscillates in period-2 cycles. We are interested in conditions under which the system converges to the optimal allocation. To this purpose we consider different incentive schemes, social norms and individual diversity.

The supervisor problem

The supervisor problem is to design a linear compensation scheme for the subordinates that induces them to use their capacity to maximize the team output. The supervisor can observe the efforts the subordinates exert with her ($u_i, \quad i = 1,2$) and the team output. Each subordinate's compensation is $w_i = s + b_i u_i + b_t (u_1+u_2)^{\alpha}(l_1+l_2)^{\beta}, \quad i = 1,2$ where s is a base salary sufficient to meet the participation constraint of the agent, while $b_i, \quad i = 1,2$ are the incentives given to subordinates for their individual effort with

supervisor and b_t is the incentive given them for team output. We assume that the supervisor declares the bonuses and that the subordinates decide their efforts in order to maximize their wage.
Theoretically, the supervisor has to solve the following problem

$$\max_{b_1,b_2,b_t} \quad (u_1^* + u_2^*)^{\alpha} (l_1^* + l_2^*)^{\beta} (1 - b_1 - b_2 - 2b_t)$$

$$\text{sub} \quad \begin{array}{l} 0 \le b_i \le 1, \quad i = 1,2 \\ 0 \le b_t \le 1 \\ (u_1^*, u_2^*) \text{and } (l_1^*, l_2^*) \text{are optimal} \end{array}$$

When considering fully rational agents insensitive to social norms the solution is obvious. Since any individual bonus given to agents gives a suboptimal effort allocation, and null team output bonus makes for subordinates any allocation optimal, the optimal solution is

$$\begin{cases} b_1 = 0 \\ b_2 = 0 \\ b_t = \varepsilon > 0 \end{cases}$$

On the contrary, when considering social norms and/or bounded rationality agents the problem becomes complex.

Norms and bounded rationality

Literature devoted great attention to social norms and peer pressure. For example, Roy (1952) documents quota restrictions in industrial workers, and documents peer-pressure in not exceeding some fixed levels of production.
In our approach these important aspects are summarized, by considering norms. In this case, individuals may have a disutility in effort allocation different from some common standards.

We assume some subordinates may be sensitive to partner and other same level subordinates behavior. In particular we consider two classes of behaviors:

1. partner's norm sensitive: these agents have a disutility in being different from their partner level of behavior and their utility function is

$$w_i = s + b_i u_i + b_t (u_1 + u_2)^{\alpha} (l_1 + l_2)^{\beta} - \delta_u (u_i - u_{3-i})^2 - \delta_l (l_i - l_{3-i})^2, \quad i = 1,2$$

2. same rank norm sensitive: these agents have a disutility in being different from the agents at their level. In other words, we assume that each agent at level k knows the average of his level agents effort $\bar{l}$ with the partner, and effort $\bar{u}$ with the supervisor. Their utility function is

$$w_i = s + b_i u_i + b_t (u_1 + u_2)^\alpha (l_1 + l_2)^\beta - \delta_{\bar{u}} (u_i - \bar{u})^2 - \delta_{\bar{l}} (l_i - \bar{l})^2, \quad i = 1,2 .$$

Norms may nevertheless be interpreted in a different way. Laland (1999) shows that for many animals adaptive behavior results in part from copying the behavior of others. "Do-what-others-do" is an effective heuristic in many situations. Animals probably do not copy other animals indiscriminately, although this has yet to be established empirically. Nonetheless, there is evidence that animals employ strategies such "Do-what-others-do" when they are uncertain, or, when there is no easier solution, they use "Do-what-the-majority-do", or "Do-what-the-successful-individuals-do".
In this sense we consider individuals that use the norm as a heuristics.

The platform

We developed a software platform in order to study some interesting aspects of the model we presented and to perform simulations of pyramidal organizations with different incentive schemes and individual behaviors. Organizations with a number of levels from 2 to 6 may be studied and parameters may be interactively modified. For instance, it is possible to observe the organization performance under with values of α and β and under different linear incentives.

The organization performance is defined the net capacity of the top manager. The theoretical performance of the organization and its inefficiencies may be monitored in real time considering the following variables:

1. *first best*: it is the output when each agent allocates optimally his capacity and no bonus is paid;
2. *second best*: it is the output when each agent allocates optimally his capacity and all the bonuses are paid;
3. *third best*: it is the net total output with actual effort allocation and all the bonuses are paid.

It is also possible to monitor single agent behavior using opportune output variables. These variables may become especially interesting when considering a two levels organization. In this case the supervisor problem is to solve a two-level programming problem.

The simulations allow the user to vary different levels of norms, both with same level colleagues and the supervisor. It is also possible to assign the agents to different classes of behaviors.

Some classes of behavior

As it is evident individual diversity play a key role in real organization. As a consequence, besides the rational behavior, we are interested in considering also bounded rationality for the agents in our model of organization. To extent different behavioral classes of behavior have been implemented. At the moment, five kinds of behavior are available, but some others are going to be introduced. In the following, we give a description of the available classes.

1. *Individualistic optimizer*: given last partner effort, this agent optimizes his effort without considering any social norm;
2. *Heuristic alpha/beta*: the relative proportion of effort between supervisor and partner is in the ratio alpha/beta;
3. *Optimizer sensitive to partner*: optimizes given last partner efforts;
4. *Norm level follower*: follows the norm of same level partner;
5. *Optimizer sensitive to level*: optimizes given last partner efforts but considers averages among society.

Some simulation results

The first simulation we present here considers norms and individual incentives. In a two-levels organization (a module) assume the elasticities are:

$$\alpha = 0.75, \qquad \beta = 0.25.$$

While agent *1* has capacity 6, agent *2* has capacity 1. When the team bonus is 0.01, the ideal total output, since there are no incentives, is 5.12889, while the net performance is about 5.0776. If operatives are sensitive to the social norm of effort provided with the supervisor ($\delta_u = 0.01$), the net total output decreases to 3.07839, while, obviously, the ideal output of the organization is unaffected. Now, assume that the supervisor increases the team bonus up to 0.1. As expected, this incentive is needed to balance the social norm, in fact, this way, the total output increases to 4.05. If the supervisor introduces in addition also an individual bonus of 0.01, then the performance has an extra improvement and it increases to 4.119938. Therefore, individual bonus that is usually detrimental, in this case can help in improving the final net output of the organization.

Conclusion and further research

The model we provide allows the study of optimal incentive scheme and optimal allocation of individual capacity in a hierarchical structure.
The simulation platform allows the extension of the results provided by theoretical analysis to multilevel organizations. The agent-based model easily allows the extension of the analysis to heterogeneous agents. In particular, the platform can well be used to study the impact that different individuals have depending on their role in the organization. Simulations that consider different kinds of incentive in order to improve non optimal situations can also be performed.

Further research is devoted in studying the impact of many other factors in the organization, such as reneging and (mis)perception of behavior by colleagues.

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