Wild@Ace 2004

Industry and Labour Dynamics II

Proceedings of the wild@ace 2004 conference

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Job Search Mechanism and Individual Behaviour

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This paper investigates upon job search mechanism at individual level by a determinstic-stochastic approach in a economy with perfect competion and rational agents. Each single unit, firm or worker, is analyzed over time; aggregate dynamics comes directly from the micro-structure of the economy. We show that the unemployment as well as the vacancy rate converge in the long run to an ergodic distribution whose average value lies on the Beveridge curve. Transitional paths are not-monotone and depending on initial conditions. The micro-model is exploited to assess the relationship between job search and social networks (neighborhood effects); results show that, when the network is endogenous, such spillovers affect both transitional paths and steady state in several way, not last in a negative way.

J.E.L. J20 - J23 - J24
Keywords: job-search, human capital, social network, epidemic processes.

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[1]I wish to thank Pierre Cahuc, Gianni De Fraja, Franco Peracchi and seminar partecipants in Rome, Milan and Turin (Wild@ACE) for useful comments and suggestions. Usual disclaimers apply.

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1 Introduction

Like most other aggregate functions in the macroeconomist's tool kit, the matching function is a black box; we have good intuition about its existence and properties but only some tentative ideas about its microfoundations. .... The most popular microeconomic models, such as the urn-ball game, do not perform as well empirically.[3]

Job search is an intrinsically individual-level issue; it sums up the behaviour of two elementary units of the economy: firm and worker. What we observe in the job market data is the consequence of such elementary processes; unfortunately we have not a clear idea on whether the aggregate level is just the mirror, on a larger scale, of the microcosm or only a fuzzy and distorted representation. Microdata shows that job market is characterized by large gross flows of jobs creation and destruction (Davis, Haltiwanger and Schuh 1996) as well as workers' transition among unemployment, employment and out of the labor force status. At an aggregate analysis, the Beveridge curve shows fuzzy movements, alternating relative stable patterns to large changes in position and slope, as well as non-monotone transitional paths (see Blanchard and Diamond, 1989, Bleakley and Fuhrer, 1997, Nickell et al. 2002). In other words, data shows that there is a remarkable level of churning in the labour market; movements of the Beveridge curve are the economy-wide response to a complex microeconomic framework. The matching function is the common thread between these two worlds.

The process characterizing the single match between firm and worker involves a high number of variables which are well reviewed in the Petrongolo and Pissarides, 2000, survey. Under this point of view, the idea of a "simple" description of the microcosm could be frustrated; nevertheless we can not investigate into the labour market without trying to challenge this point. This paper presents a first attempt at providing a simple but reasonably descriptive analytical body for modelling job search at the individual level which is coherent with a simple macroeconomic model. It relies heavily on recent developments of mathematical and statistical research applied to the investigation of the dynamics of epidemic processes.

The simplest way at looking the job search mechanism is the one described by the Pissarides Mortensen approach (PM hereafter): at every time a firm randomly "contacts" a worker, the match is closed and the unemployed worker changes her status to employed (hence she is removed from the unemployment population), the firm fills the vacancy and it can be removed from the population of vacant firm. Meanwhile, new unemployed are born and new vacancies are opened as the result of an exogenous separation between an existing firm-worker unit. The matching rate, as well as the separation one, can depend both on worker features, notably human capital, and firm characteristics as well as on local spillovers.

However, before of describing the search and matching process, we have to identify properly the actors of this play. As pointed out by Blanchard and Diamond, the common view of considering workers either as unemployed or employed is too narrow: *Much of the movement into ad out of employment is from out of the labor force*[4] and again *the relevant pool of workers appears to include some workers classified as being out of the labor force*[5]. Persons not in the labor force who want a job now is the

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[3] Petrongolo and Pissarides, 2000.

[4] Blanchard and Diamond, 1989, page 3.

[5] Idem, page 4.

standard U.S. Bureau of Labor Statistics definition of this share of workers and in general is a form of hidden unemployment or labour reserve (see Castillo 1998 for a detailed description). According to the US Current Population Survey, in September 2003 the number of workers who were out of workforce not looking for a job but that would like to work was about 4.6 million; it is worth stressing that the larger share of hidden unemployment lies between age 16 and 54, i.e. in the most productive part of the working life. Average duration of hidden unemployment is about 12 months in the CPS; nevertheless this group of workers do not show a higher labour market attachment. Following the Castillo analysis, *only 41 percent of nonparticipants who said they wanted a job in 1994 were in the labor force in 1995* as opposed to the strong labour attachments shown by traditional unemployed job-seekers: 78 percent continued to be labor force participants in 1995. The hidden unemployment is a sort of temporary retirement from the job market and this choice depends on a lot of individual characteristics not least on the discouraged worker effect; according to the US BLS: *Discouraged workers are those persons who say that they want a job, were available to work, had searched for a job some time in the previous 12 months but had stopped looking for work because they believed that there were no jobs available for them.*

Once they decide to become job-seekers, persons out of the labor force have more difficulty in finding a job. This introduce a difference in the duration of unemployment between these two groups, which broadly we can define long and short term unemployment[6]. In general the distinction between attached and unattached workers is crucial for the analysis, as Blanchard and Diamond show, and both theoretical and empirical models should account for it.

Another relevant point concerns quits and layoffs with or without recall. Workers who leave voluntary the job are usually considered as retired from the labour force or not available for a "new hire", at least temporarily. Voluntary quits can also be driven by an employment-to-employment reason but even in this case this flow does not create new hires; Akerlof Rose and Yellen, 1986, calculate that 40 percent of workers quitting their job move directly to a new job without enter the unemployment status. More controversial concerns layoffs: *A worker who is laid off may remain attached to the firm in two distinct senses. One is that the worker is less available for employment elsewhere than the typical unemployed worker. The second is that the worker is available for recall by the firm without the need to post a vacancy*[7]. Moreover, Blanchard and Diamond assume that even the attached fired worker becomes unattached over time if not recalled or hired in another job. These points induce econometricians to exclude job losers on layoff from the stock of unemployment used to estimate the matching function but including persons out of the labor force but that would accept a job if offered, for the reason previously recalled.

Our paper tries to set-up a theoretical model of job search and matching on individual level providing also dynamic equations for the economy as a whole. The questions recalled so far induce to believe that the standard dichotomy between employed and unemployed of the standard MP approach is too stringent and it should be relaxed. On the other hand the introduction of either element makes quite hard the development of an analytical body. By the point previously discussed it is possible to infer two questions: the importance of the introduction of persons not in the labor force who would a job in the total labour supply and the explanative hierarchy between job-losers and job seeker for the matching

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[6]Blanchard and Diamond find that long-term unemployment is a good proxy for the pool of workers out of the labor force (page 32).

[7]Blanchard and Diamond, ibidem, page 18.

process. This twofold remark coming from current literature induce us to consider labour force as being composed by unemployed job-seekers, U , employed, E , and out of the labour force or unattached, R. It is worth stressing that this status can proxy for the long-run unemployment, as pointed out by Blanchard and Diamond. Nevertheless we have also to identify how workers change their status; this is particularly important for the probabilistic model we will see in section 5.
By accounting for three workers status, all possible transitions are:

$$UU, UE, UR, EE, ER, EU, RR, RU, RE.$$

Table 1 shows monthly average of job market gross flows, in percentage, as recorded by the U.S. Labor Office, for both sexes:

	UU	UE	UR	EE	EU	ER	RR	RU	RE
2000	42.3	32.4	25.3	96.0	1.1	2.9	92.8	2.1	5.1
2001	46.2	29.9	23.9	95.6	1.3	3.1	92.6	2.3	5.1
2002	52.6	25.2	22.2	95.7	1.4	2.9	92.7	2.5	4.8
2003	53.3	23.9	22.8	95.8	1.3	2.9	92.7	2.5	4.8
2004 (Jan./Apr. 04)	53.3	23.9	22.8	95.8	1.3	2.9	92.7	2.5	4.8

Source: U.S. Labor Office

Table 1 - US gross flows

Although data are not adjusted for the business cycle, it is rather surprising the amount of workers leaving the U status to enter the R one. Nevertheless this workers flow is not relevant for the search and match process, as they do not fill a vacancy; for such a reason we are not interested in modelling this share of workers in the paper.
When fired the worker has to perform a choice whether to join again the job market (EU) or to go out from the labor force (ER). According to the data, the flow EU is negligible in magnitude and lower than the ER one. Fired workers have to be considered carefully. We have previously recalled that lay off involve either strong attachment to the firm, for example because of firm or industry specific human capital, and in this case we are close to the employment-to-employment flow, or a low attachment. Blanchard and Diamond suggest to consider low attachment as a form of long term unemployment, opposed to the short term one characterizing workers in the U state.
Finally the state R . Data shows that workers in such a state have a very low mobility: about 93% of them stay in this state during 2000-2004. They leave R either to be employed or to get a job-seeker. In both cases this worker fills a vacancy but following two different ways. In the first one she enters directly the E status without passing through the U state. This involves two matching function in the model: one driving the RE flow and another the UE one.
In order to build a theoretical model we have to undertake some simplifying assumptions with respect to table 1. First of all, as said, we are not interested in modelling the UR flow. Moreover, we want to focus on the matching function as a device linking individual transitions to the aggregate rate of vacancy and unemployment, as both theoretical and empirical literature points out; under this point of view, the mechanism driving the RE flow involves a second matching function that is not relevant for the debate on job search. At the same time, we do know that a fraction of R workers apply again for a job. In order to make the model analytically tractable and at the same time sufficiently detailed, we assume that the flows RE and RU can basically be represented by a single transition RU ; in other words, after a lay-off, workers are in the state R and have a chance to be matched again via the transition to the U status. Finally, we neglect the EU flow since it refers to a small share of fired individuals.

In such a way, individuals can be hired only by passing through the U status; we can think to U as a sort of job center: any worker interested to a job vacancy must apply herself by joining the U status. In such a way we need only a matching function driving the UE flow and we shall show that such function obeys to the standard requirements of the MP approach.
The second main point of the paper is to provide a mircrofoundation to the matching function. At this end we have to introduce some individual component driving the transition from U to E, leaving exogenous the separation rate related to ER and the one supporting RU. Our idea is to link the chance to be hired to some form of search intensity.
Our goal is hence to provide an alternative approach to the job search issue based on a probabilistic law that allows us to analyze coherently both the single unit history, i.e. the micro-dynamics, and the economy as a whole.
We will do this by building a probabilistic model describing individual behaviour over time. Moreover search intensity, driving the transition rates $U \to E$ and $V \to F$ - where V stands for vacancy and F for filled - is made endogenous by a rational behaviour. From the micro-model we also obtain the differential equations describing average transitional path to the long run equilibrium - i.e. to the Beveridge curve - providing the dynamical analysis over the economy as a whole, an investigation close in spirit to the model developed in Blanchard and Diamond.
In the last part of the paper we focus on how transition rates are affected by local spillovers or social networks. A part of job matches come from alternative channel than the usual posted vacancy; as recalled in Cahuc and Fontaine, 2002, " *a large proportion of people (about 50% on average) hear about or get their job through friends and relatives*". We will investigate on transitional path and steady state when the worker transition rate $U \to E$ depends in part on the worker search intensity and in part on her social network. In particular we are going to assume that the transition rate increases according to the number of employed person in a surrounding of the individual. This a typical spillover effect, close in spirit to the literature focussing on local interactions in the accumulation of human capital. It is worth stressing however, that, unlike current literature, our probabilistic model allows us to make endogenous the network structure, in sense that the number of employed workers changes over time according to the matching process of the economy. This has remarkable effects on results; social network can or not increase matching efficiency according to the weights put on the local component of the transition rates, parameters and initial conditions. A reduction in efficiency is possible when the matching process relies heavy on the local component and the social network tends to be emptied over time, i.e. the number of employed individuals reduce as a consequence of an increase in unemployment. This result induce more caution in stressing the importance of local effects on improving job matching than the one now recognized.
The paper is organized in four logical steps: the first one is devoted to the individual analysis where "optimal" transition rates are obtained according to a rational behaviour of firm and worker. The second one shows how obtaining macro equations by averaging over these individual rates. Subsequently the probabilistic model shows how we can obtain the micro-dynamics coherently with the macro one. In the last section of the paper, we show how the probabilistic micro-model allows to take into consideration "local" or "social network" components in the job search mechanism.

2 Single Worker Analysis

We start by considering the individual working life. As pointed out in the previous section, we are going to divide the total labour supply in job seeker, U, employed, E, and unattached or long run unemployed, R. Obviously $L = U + E + R$. Total labour supply L provides the metric for the flows hence $1 = u + e + r$ where $x = X / L$. K is the total number of firms in the economy; when $L = K$ full employment is possible, since we are going to assume, as in the MP spirit, one worker one firm. In such a case $F + V = L$ or $f + v = 1$ where obviously $f = e$.

A worker randomly drawn from the population occupies one of these three status. The status are hierarchically ordered: a worker is employed only after she was a job seeker, i.e. $U \to E$. For reasons previously recalled, we assume that once the worker separates from the firm she does not became immediately a job seeker but stays in the state R before entering again the state U; the average time of occupancy of the status R depends obviously on the transition rate from state R to U. As will be clearer later, this assumption of stages hierarchically ordered allows us to characterize the steady state of the job market as a balancing of flows out and in the U, E and R states; in this way the model fits the official partition of the work force in unemployed, employed and labour reserve.

Under a probabilistic point of view, the simplest way at modelling the single worker lifetime is a "three states process". Worker lifetime is a continuos jump in and out three possible states: U, E and R. Each agent is characterized by an own string of U, E and R recording her history over time; the length of time the individual stays in a given state is a random variable and the Poisson distribution is a good approximation of this occupancy time, as well known.

This process can be easily described in a probabilistic way: if the worker is unemployed but actively engaged in job search at time t, she can change her status to employed at the instantaneous rate α, and the probability that she will be effectively employed in the time interval $t + \Delta t$, is simply $\alpha \Delta t$. Likewise, when the worker is employed, she has a probability $\beta \Delta t$ to be separated during $t + \Delta t$. Finally an unattached worker has a probability $\mu \Delta t$ to be a job seeker during $t + \Delta t$. The coefficients α, β and μ are the transition rates; they can depend both on individual and firm characteristic, although we are going to treat them as exogenous parameters along this section.

When the worker changes her status, she stays in the new state for a random time $\mathcal{E}$ as said, this waiting time is well modelled by a Poisson process. Our worker is a job seeker for a given time T_1^U after then she becomes employed for another random time T_1^E then changes again to unattached for a time T_3^R and finally come back to the status of job seeker and so on. The dynamic of the probability to be in a given state at time t follows the Kolmogorov forward equations (see Cox and Miller, page 172):

$$\frac{dP^E(t)}{dt} = -\beta P^E(t) + \alpha P^U(t)$$

$$\frac{dP^U(t)}{dt} = -\alpha P^U(t) + \mu P^R(t)$$

$$\frac{dP^R(t)}{dt} = -\mu P^R(t) + \beta P^E(t)$$

with initial condition and $P^U(0) = 1 - P^E(0) - P^R(0)$ given. The above equations have a prompt interpretation; they describe the evolution of the probability over time according to a balancing of

inflows and outflow. As an example, let us focus on the first one; the probability of finding a worker in the employed status increases with the inflows from unemployed to employed $\alpha P^U(t)$ and decreases with the separation $\beta P^E(t)$.
The steady state solution is :

$$P^E = \frac{\mu\alpha}{\beta\alpha + \beta\mu + \alpha\mu}$$
$$P^U = \frac{\beta\mu}{\beta\alpha + \beta\mu + \alpha\mu} \qquad (1)$$
$$P^R = \frac{\beta\alpha}{\beta\alpha + \beta\mu + \alpha\mu}$$

The steady state is a stable node and so the long run distribution does not depend on initial conditions; this means that if we observe a single worker job history over a sufficiently long horizon, then the percentage of time spent in a given state, E , U, or R is close to the theoretical distribution. The strong convergence and the independence of initial conditions are valuable properties for empirical analysis.
From here we have the following lemma:

Lemma 1: *Over a sufficiently long time, the proportion of time spent in any state* $i \in \{U, E, R\}$ *converges in probability to* P^i, *where* P^i *is given by eqs. (1).*

Proof: See Cox and Miller, 1972, pages 172-175.

This lemma is particular useful to our scope; it says that if the worker lifetime is sufficiently long, as we are going to assume, then the individual knows the expected time of being employed, job seeker or out of work force . This property will be exploited to analyze individual behaviour.
If the worker lifetime would consist of two states only, U and E, then $P^E = \frac{\alpha}{\alpha+\beta}$ and $P^U = \frac{\beta}{\alpha+\beta}$; Blanchard and Diamond, page 7, call P^E as c, for cycle, in sense that it measures the degree of aggregate activity in steady state and $s = \beta c$, where s stands for shift, as an index of the intensity of reallocation in the economy.

3 Individual Choice

In the previous section we have modelled the transition probability for individuals as driven by exogenous parameters α, β. and μ. Now we are interested in modelling the transition probability from job seeker to employed; this entails a sort of "search effort". In this section we are going to assume that, conditional on the U state, the worker has to choose an optimal search effort before joining the job market. In other words, workers choose α optimally, leaving β and μ exogenous. As previously stressed, individual lifetime is a continuos change over three admissible status; Lemma 1 says that the expected time spent in each state is given by the probabilities entailed in eqs. (1). Before being hired, the worker is obviously in the state U, looking for a job. She has to invest in the search activity which is costly in effort term; aim of the search is to be hired and rewarded with the wage.

When employed the worker earns w per unit of time[8], while as job-seeker she incurs in a instantaneous search-cost $C(\alpha)$ proportional to the search effort; the higher α, the higher the transition probability from U to E but the heavier the effort. Finally we assume that when the worker is out of the work-force she earns neither w nor pays for the search cost $C(\alpha)$; in particular we simplify assuming zero value for this state.
The present value of the expected income stream conditional on the U state is given by the following equation:

$$V^U(\alpha,t) = -C(\alpha) + \int_t^\infty e^{-\rho\tau}\left[(P^E(\tau)w - P^U(\tau)C(\alpha)\right]d\tau$$

Perfect competition drives $V^U(\alpha,t)=0$, since the rent will be totally exploited by workers[9]. Solving explicitly equation value V is impossible; however a remarkable simplification is achieved by using long-run probabilities given in eqs. (1). This approximation is not crucial for results since convergence of $P^i(\tau)$ to P^i is rather fast; if the reader prefers, we are performing a local analysis over the steady state.
By assuming $C(\alpha)=\alpha^2$, simple algebra leads to:

$$\alpha_e^* = \frac{\sqrt{\mu\left(\beta^2\mu(1+\rho)^2 + 4w\rho(\beta+\mu)\right)} - \beta\mu(1+\rho)}{2\rho(\beta+\mu)} \qquad (2)$$

Optimal effort is increasing in w. The supply curve in the economy is upward sloping: the average number of matches is $\alpha_e^* U$ and is increasing in w.
As far as firms is concerned, they have to choose the optimal search intensity α_f, i.e. the transition probability per unit of time of jumping from vacant to filled. In this case the MP approach applies: the Bellman' equations for, respectively, the filled V^F and vacant V^V status are:

$$\begin{aligned} rV^F &= y - w + \beta(V^V - V^F) + \dot{V}^F \\ rV^V &= -c(\alpha^f) - \alpha^f(V^V - V^F) + \dot{V}^V \end{aligned}$$

Perfect competition requires $\dot{V}^V = V^V = 0$ hence $V^F = \frac{c(\alpha^f)}{\alpha^f}$. By assuming a quadratic search cost we have that the firm value when filled coincides with the search intensity $V^F = \alpha^f$. As in Pissarides (pages 28 and 29) the only rational expectation equilibrium requires $\dot{V}^F = 0$ and this brings immediately to:

$$\alpha^* = \frac{y-w}{r+\beta} \qquad (3)$$

[8]Workers are wage takers.

[9]The same reasoning holds in the standard MP approach when the rent of posting a vacancy, V^V, is driven to zero by perfect competition among entrepreneurs.

the higher the wage, the lower the search intensity. In this case the demand curve is downward sloping. Wage determination comes from a market clearing condition instead of a bargaining process, as in MP; instantaneous market equilibrium requires that same number of matches comes form both sides of the market, i.e. $\alpha^* V = \alpha_e^* U$ or $\alpha^* v = \alpha_e^* u$ at the same wage w. By solving (2) and (3) jointly with the market clearing condition $\alpha^* v = \alpha_e^* u$, we obtain:

$$\alpha_e^* = \frac{\sqrt{\phi_1 u^2 + \phi_2 uv + \phi_3 v^2} - \mu\left(2\beta v + u(r+\beta)\right)}{2v(\mu+\beta)} \tag{4}$$

where $\rho = 1$ for simplicity and $\phi_1 = \mu^2(r^2 + 2r + \beta^2)$, $\phi_2 = 4\mu^2\beta(r+\beta)$, $\phi_3 = 4\mu(\beta^2\mu + \mu + \beta)$. It is straightforward to see that α_e^* is always positive.

Given the individual transition probability, the average instantaneous matching function for the entire economy is $m = \alpha_e^* u$, i.e.:

$$m \equiv \alpha_e^* u \equiv \alpha^* v = \frac{\sqrt{\phi_1 u^2 + \phi_2 uv + \phi_3 v^2} - \mu\left(2\beta v + u(r+\beta)\right)}{2(\mu+\beta)}\frac{u}{v} = m(u,v) \tag{5}$$

Although a bit cumbersome, the matching function is increasing, concave and linearly homogenous of degree one in u and v, as in the spirit of the MP approach.

4 Macro-Dynamic Analysis.

The model is essentially dynamic in its nature and in this section we are going to obtain the differential equations characterizing the average dynamics; given an initial condition, the job market evolves towards the steady state represented by a point on the Beveridge Curve.

Given the instantaneous transition rates we derive the differential equations for the economy by averaging over firms and workers. For a job seeker, the average probability, per unit of time, of changing status from unemployed to hired ($U \to E$) is α_e^* times the probability of finding an individual in the unemployment status, P_U, i.e. $\alpha_e^* P_U$, where $P_U = U/L$. In the time interval Δt, corresponding to L elementary steps[10], the average number of unemployed being employed is $\alpha_e^* P_U L \Delta t$; hence, the number of unemployed decreases at the rate $\Delta U / \Delta t = -\alpha_e^* P_U L = -\alpha_e^* uL$. The number of unemployed instead increases by μR. Likewise the number of vacancy decreases at the rate $\Delta V / \Delta t = -\alpha^* P_V L = -\alpha_e^* \frac{u}{v} P_V L = -\alpha_e^* uL$. Clearly the average matching function m must be the same both for unemployed and vacant firm since that, once a match is closed, both variables decrease by one unit as we assume, as in MP, one worker - one firm. Conversely the number of vacancies increases, in the unit of time, when a separation occurs, i.e. $\beta(1-u-r)$. Finally the number of unattached workers increases by $\beta(1-u-r)$ and decreases by μR.

Summing up, we obtain the following dynamical equations:

[10]For computational purposes we normalize the number of elementary units to the size of the economy.

$$\frac{du}{dt} = -m(u,v) + \mu r$$
$$\frac{dv}{dt} = -m(u,v) + \beta(1-u-r)$$
$$\frac{dr}{dt} = \frac{dv}{dt} - \frac{du}{dt} = \beta(1-u-r) - \mu r$$
$$u(t) + e(t) + r(t) = 1$$
$$e(t) + v(t) = k$$

By imposing the steady state condition $\frac{dv}{dt} = 0$ in the second equation and substituting from the third we obtain the long run relationship between vacancy and unemployment, i.e. the Beveridge curve.
The model has three equations in two unknowns, since $r(t)$ is obtained by the previous two, leaving the dynamics essentially driven by $u(t)$ and $v(t)$; the determinant of the Jacobian matrix is zero and there are two complex roots with a negative real part while the third is zero. In such a situation, all solutions converge to a three-dimensional manifold with the consequence that the steady state, as well as the transition path, depends on initial conditions (see for example Kamien and Schwartz page 347). The Beveridge curve is the projection of such a manifold in the bidimensional plane u, v . This makes impossible to analyze dynamics by the standard phase-diagram technique and numerical solutions must be carried out.
In order to calibrate parameters, we are going to use data by the US Labour Bureau, and in particular by the recent monthly Job Openings and Labor Turnover Survey (JOLTS)[11]; unfortunately the survey does not cover period before December 2000. Data on unemployment comes from the national unemployment rate seasonally adjusted. Figure 1 plots the monthly series trough January 2001 and November 2003 jointly with and estimated tendency line.

[11]See Monthly Labor Review, December 2001, for details.

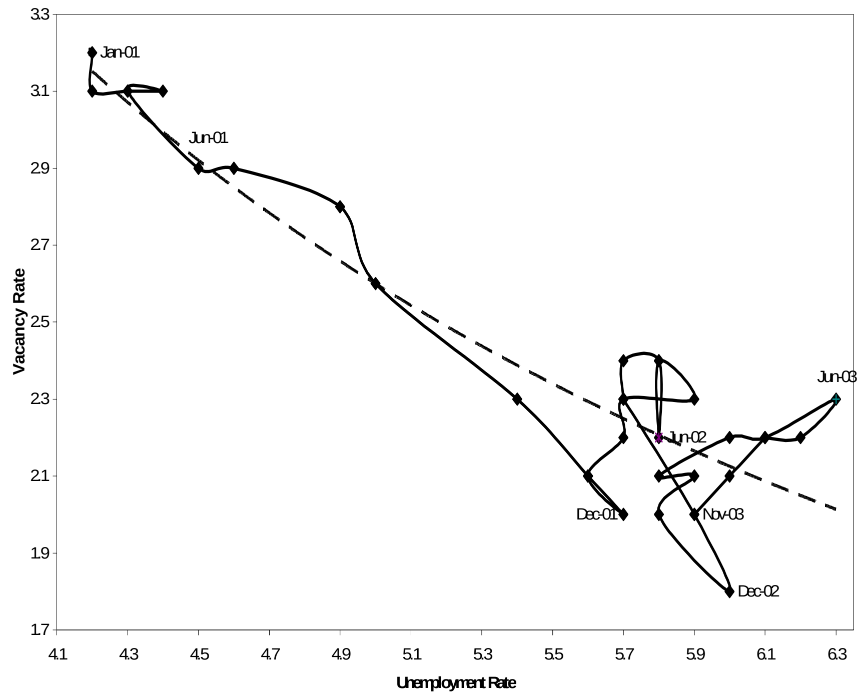

Figure 1: U.S. Beveridge Curve

Although the figure is quite fuzzy, during 2001 the inverse relationship between v and u is rather stable. From January 2002 to November 2003 the curve undertakes some rapid fluctuactions around the tendency line.

We calibrate parameters on the estimated beveridge curve of Figure 1 where the unemployment rate spans between 4.2 and 6.3 percent while the vacancy rate between 3.2 and 2.0 percent. Figure 2 shows the calibrated Beveridge Curve and some transitional paths.

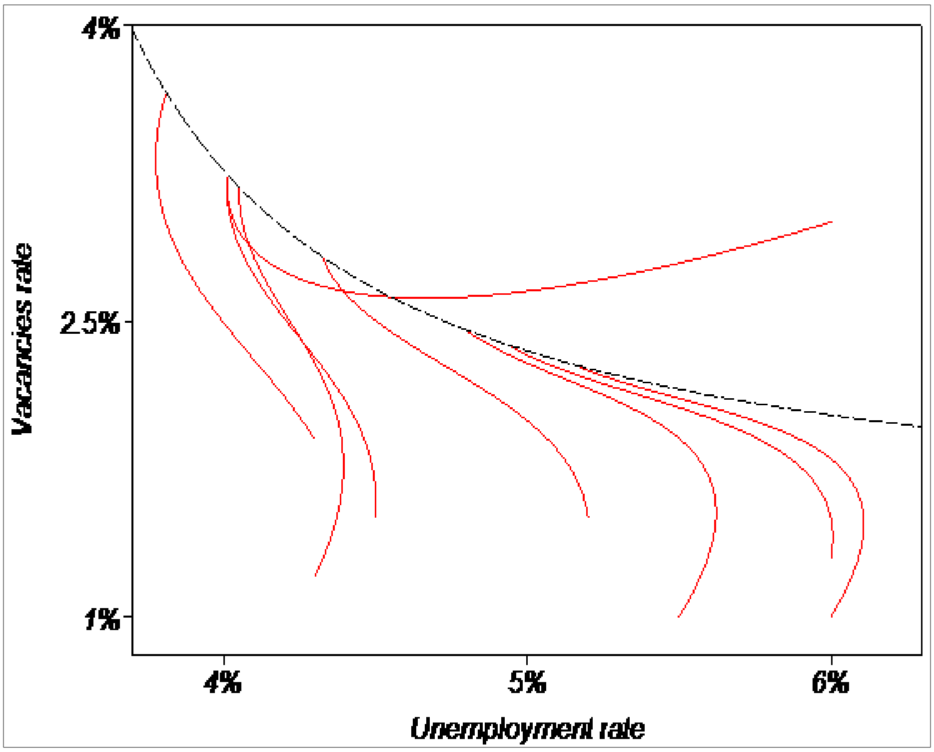

Figure 2: Calibrated Analysis

Transitional paths can be either clockwise or anti-clockwise depending on initial conditions; in general starting from a vacancy rate below 1.9% and unemployment higher than 5.4% involves anti-clockwise dynamics. Vacancy higher than 1.9% involve clockwise dynamics whatever the initial unemployment rate. There is an intermediate parameters region, broadly $v \leq 1.9\%$ and $u \leq 0.54$ where the transition path follows a "S" path.

This sort of path-dependency of the system is interesting in the comparison of the working of the job market and equilibrium unemployment across countries: long run differences are induced not only by a different institutional setting or search mechanism but by different initial conditions as well. It is worth stressing that a similar idea can be found in Blanchard and Diamond, although they do not formalize it; in particular they stress that in a real job market movements into and out of unemployment are more complex than the standard job search model, since these flow not necessarily involve posted vacancies (this is the case of strongly attached workers). In this case: *what happens to vacancies and unemployment after a shock will depend on the initial stocks of attached and unattached workers, which themselves will depend on the history of the shocks.....Whether aggregate activity shocks generate counterclockwise movements in the Beveridge space is much more ambiguous*[12]. Although these authors points out the shock response of the Beveridge curve with the consequent transitional paths, they observe that the distinction between attached and unattached workers make the dynamics dependent on past history. The same holds for our model; the type of transitional path is strongly

[12]Blanchard and Diamond, 1989, pages 19-20.

affected by the initial condition on the R status, i.e. the one modelling unattached worker as opposed to the strong attachment shown empirically by job-seekers.

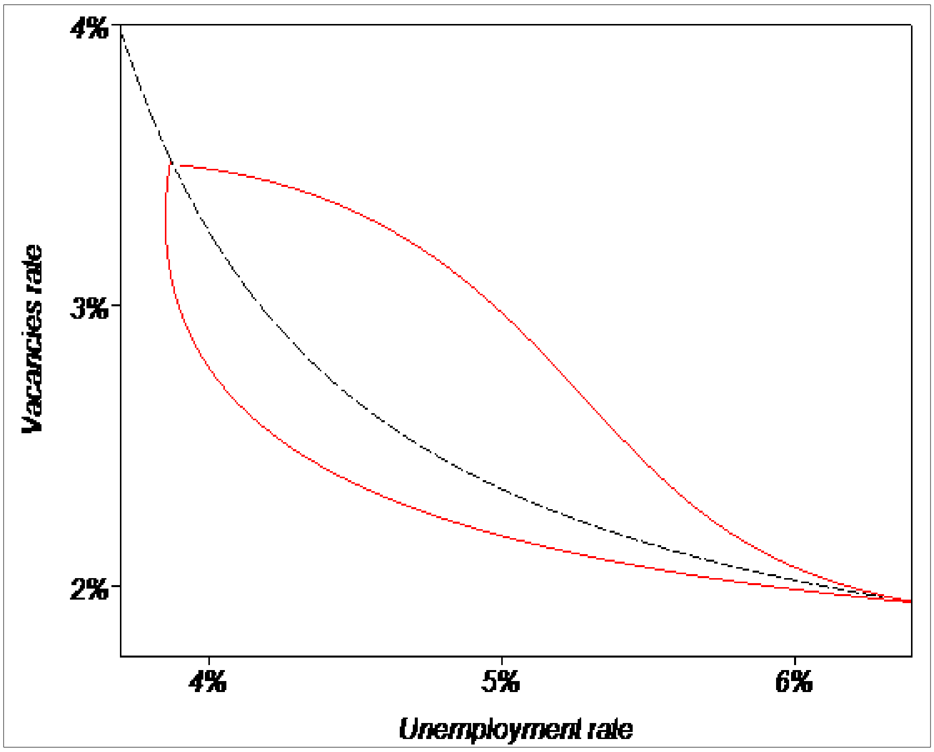

Figure 3: Spurious Cycle

Although the dynamical model does not allow to obtain endogenous cycles around the Beveridge Curve, it is possible identify proper initial conditions and parameters producing dynamical paths that behave much like as cycle. Figure 3 shows this type of spurious cycle; the dependence on initial conditions makes hence rather hard to distinguish, in real data, a genuine cycle from a shift in parameters.

This richness in transitional dynamics holds also in response to a parameter shock; Figure 4 shows a simulation where β increases by 20%. As expected, the Beveridge Curve shifts to the right reducing matching efficiency; the adjustment is counterclockwise but other paths are possible according to the initial conditions, as previously stressed. This type of exercise is close in spirit to Blanchard and Diamond figure 2; an increase in β in our model corresponds to a shock on the shift s parameter measuring the intensity of reallocation. However, unlike these authors, our model provides richer dynamics and, as said, counterclockwise adjustment is possible but it is not the only case.

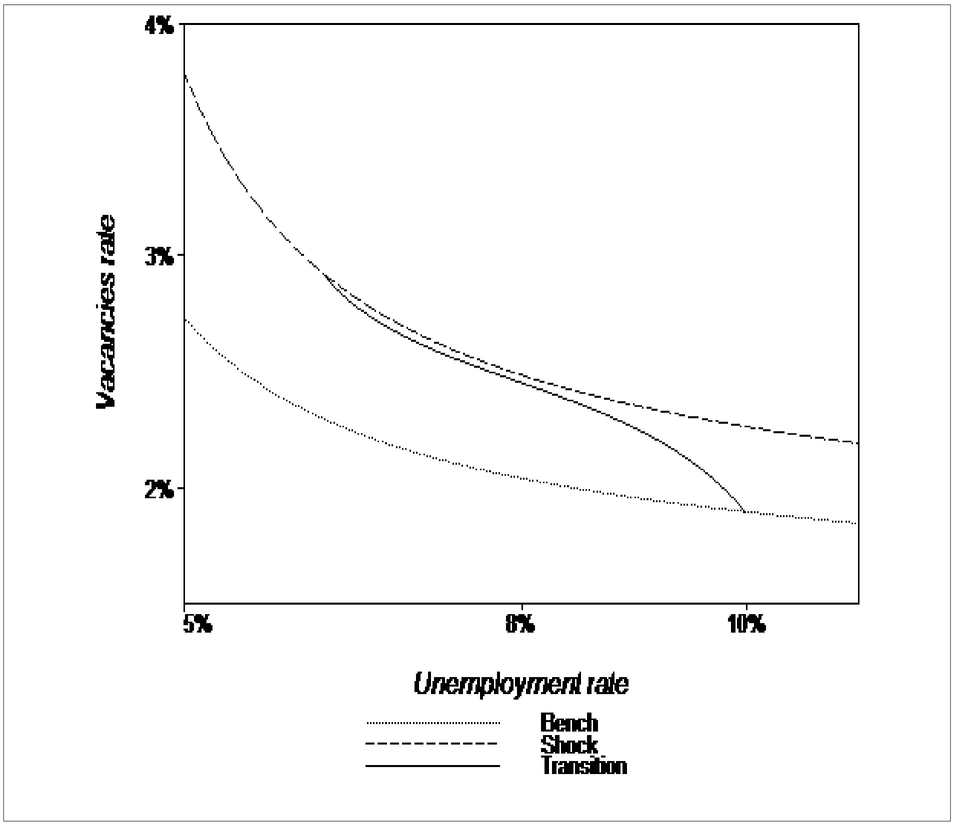

Figure 4: Shock Response

More complex transitional paths can be obtained adding to the matching function a temporal trend and/or some trigonometric function capturing waves and cycles[13]; nevertheless this is out of our scope and we do not go further on this.

5 Micro-Dynamic Analysis

In the previous section we have obtained the dynamical equations describing the behaviour of the averages (macro-variables) over time. In this section we show the microeconomic behaviour driving the macro one. In other words, we are going to set-up a microeconomic world whose averages behave exactly as the equations obtained in the previous section; if both ways of obtaining the average dynamics produce same results then we conclude that the macro and micro-analysis are coherent to each other. In this case we can indifferently investigate both the macro-world and the life history of a single agent, depending on our research goal.

Probabilistic approach at modelling our problem involves, theoretically, the analysis of the Kolmogorov equations, as we did for a single agent. Unfortunately the dependence of the transition rates on u and v makes very hard or impossible managing it. This type of problems are common to other disciplines making use of a probabilistic approach to systems characterized by a large number of

[13]We do not show such simulations for brevity.

states. Recently epidemiologists working on the spreading of diseases across individuals have exploited a dynamic Monte Carlo algorithm which can be fruitfully applied both to the numerical solutions of the probabilistic models used to forecast the evolution over time of diseases and to the estimation of individual transition rates; a good description of these methods is in Gilks et al., 1996. In particular we are going to follow the Monte Carlo algorithm presented by Aiello et al., 2000 and 2001. For a technical and detailed description of the algorithm and its probabilistic features the reader should refer to the original papers; here we present the main steps of the methodology applied to our problem of job search.

We sum up the model basic assumption:

1) Workers amount to L and firm to K and the technology is 1:1, so full employment is theoretically possible when $L = K$. At the beginning, workers are divided in three sub-populations: employed E_0, unemployed looking for a job U_0 and long run unemployed or unattached workers, R_0. Firms are divided in vacant V_0 and filled F_0. If $L = K$ then $U_0 + E_0 + R_0 = L = V_0 + F_0$ and $F_0 = E_0$.
2) Workers and entrepreneurs are completely separated and there is not flow from one to other; we can assume that this partition of the population is based on different abilities, managerial skills, business risk aversion and so on. The population is stationary.
3) When an unemployed worker meets successfully a vacant firm they become a productive unit labelled by the status M. Matched units can separate at the exogenous rate β. After separation, the worker enter the state R - she comes out from this state as a new seeker at the exogenous rate μ.− and the firm becomes V. By doing so, workers have three possible stages hierarchically ordered, i.e. $U \rightarrow M \rightarrow R$ whilst the firm has only two $V \rightarrow M$.
4) As in MP, firms and workers do accept the match, no matter who is the proponent; we are interested in counting the number of matches and separations over time, and updating the set $\{U, M, R, V\}$ consequently, and it does not matter who is "driving" the match. There is a strict duality between workers and firms spaces.
5) Transition rates from U to M and from V to M are given by eqs (3) and (4). From M to R and M to V by the exogenous separation rate β. Finally the exogenous parameter μ is the transition rate from R to U.

The DMC algorithm allows us to investigate this structure over time. As stressed at point 4, it does not matter who is leading the matching process; what is important is that when a match occurs in the economy, the sub-populations are adequately updated. Since we are interested in analyzing the search mechanism on the side of the individual worker, we are going to focus on this population. Workers are distributed into three sub-populations initially randomly allocated in a one-dimensional vector; workers are hence represented as a random collection of indexed individuals $U, U, M, R, M, U...$ on a one dimensional space. The dual vector representing firms is $V, M, M, V, V, M....$.

1) At each elementary time step l one individual is randomly drawn from the population; let us suppose an unemployed worker. This agent could change her status from unemployed to matched according to α_e^* but from a probabilistic point of view there can be other agents in the economy whose probability of changing the status is higher than the one characterizing our individual. So in deciding whether effectively the drawn individual will undertake a change, we have to compare her transition rate with the higher one in the population, i.e. $p_U = \alpha_e^* / W_{Max}$ where $W_{Max} = \sup P, .P = \{\alpha_e^*, \beta, \mu\}$. The method suggested in Aiello et al., 2000, is to compare p_U against a random number r in the closed unitary interval taken from a uniform distribution; the uniform assumption models a situation where every agent has the same a-priori probability of a

jump; this means that when $p_U < r$ the real situation is less informative than the one characterized by a total ignorance about individual transition rates. Hence, if $p_U > r$ the drawn individual changes her status otherwise she stays in the original one. When the individual changes the status the sub-populations U, V, M, R are consequently updated.

2) The waiting time between two consecutive jumps follows a Poisson process at rate $r = \alpha_e^* U + \beta M + \mu R$.
3) After the delay time, a new draw is performed and the simulation restarts.

The result of this Markov Chain[14] is the convergence of the sub-populations to a long run probability measure whose first moment is given by the deterministic equations odes. In the following section we show some simulations.

6 Results

We have performed 20 replications of the Monte Carlo method described in the previous section; we have chosen 20 runs because is enough to obtain a smooth curve to be compared to the deterministic solution. Figure 5 shows the adjustment path obtained by the average equations and the average of 20 simulations obtained by the micro-model[15]. As the reader can see, the fit is satisfactory[16].

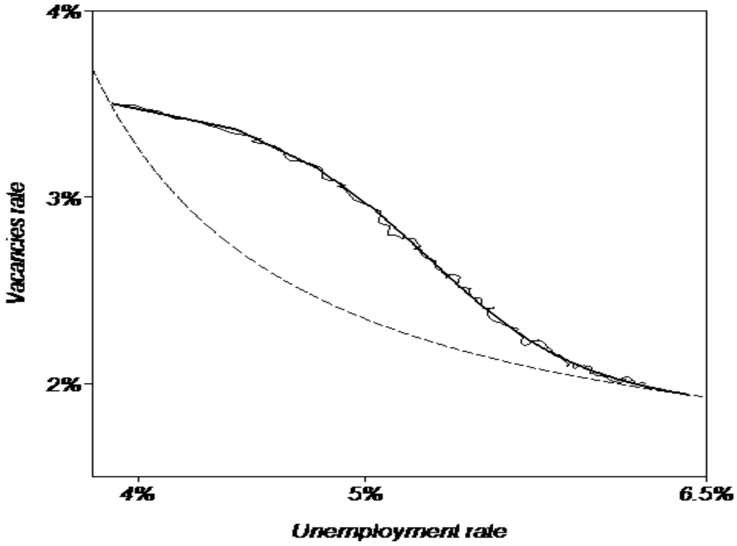

Figure 5

By doing so, we can describe the adjustment path to the Beveridge curve indifferently by the macro-

[14]The dual DMC is the one where we analyze firms instead of workers but the two chains lead obviously to same results.

[15]We plots only one transitional path for brevity, being the numerical computation highly consuming in time. For this reason we graph a transition path broadly following US data in 2001.

[16]A closer and smoother curve can be obtained by averaging over a higher number of simulations. Nevertheless this is very time-consuming.

equations and the microeconomic model; both analytical approach provides same results. Under this point of view, the macro model is well founded in the microstructure. The probabilistic law characterizing the micro-behaviour is working properly: it does provide same dynamic path of the deterministic rules driving the first moment of the distribution.

7 Local Effects and Individual Behaviour

Once built a suitable analytical tool that allows us to obtain average dynamics from the micro-structure, it can be used in a very flexible way to modify the individual behaviour. In this section we show how we can account for local interactions in the search mechanism. It is not hard to assume that matching can depend on "local" variables. Local interactions, or social networks, are often invoked in the human capital literature for explaining "club convergence" and inter-generational inequality persistence. In the search model social networks are analyzed in Calvo-Armengol and Zenou 2001, Cahuc and Fontaine, 2002 where the social network is used to spread the job offer into the network, reducing the individual searching cost.

These externalities are interesting not only because they modify the economic behaviour but mainly because they introduce heterogeneity among agents; as it will be clear, in this section individuals are distributed according to their own transition rate; such an effect allows us to investigate upon workers social mobility and the endogenous arising of "clubs" or unemployment traps.

The way the literature accounts for local spillovers is by adding complementarities between individual and aggregate variables; this approach is followed in this section. The natural candidate at representing the aggregate variables is obviously the matching function, as it is related to the congestion of the whole job market through the u and v variables. But crowding in the job market is important at a local level too; individual job search mechanism can be heavily affected by the functioning of the local market or, simply, by the number of unemployed or employed people characterizing the local socioeconomic context where the agent lives. To account for this local congestion effect, we are going to assume that the individual matching rate depends both on aggregate variables and on the number of workers in a neighboring of the unemployed individual (social network): the higher the number of working agents in a surrounding of the unemployed worker, the higher the matching rate. We can think to a sort of complementarity effect or to a congestion effect working in a positive way; the lower number of unemployed is in general positive for two reasons: it is a signal of a "good" state of the job market and, as in Calvo et al., this provides a better spreading of the job offer at a local level.

Let us define p as the probability that an unemployed worker becomes working due to the presence of a worker in the neighbor and so $(1-p)^n$ is the probability, per unit of time, for a unemployed not to become working if she has n working neighbors; consequently $(1-(1-p)^n)$ is the probability, per unit of time, for a unemployed to be hired if she has n working neighbors. When n is large the local term tends to 1 whilst when $n=0$ the local term is zero (no-spillover); an increase in n makes higher the transition rate from U to M. The individual transition rate modify in:

$$U \to M \quad : \; W_U = \Gamma\alpha^*(u,v) + \Lambda(1-(1-p)^n)$$

where $\Gamma + \Lambda = 1$ in order to balance the local effects. Under this point of view, homogeneity of degree one for the matching function involves only the microfounded term since the local effect is a pure externality. It is worth stressing that, unlike Calvo-Armengol and Zenou 2001, Cahuc and Fontaine,

2002[17], in our approach n varies across workers and over time; this implies that there is a distribution of matching rates at each time t. This characteristic is allowed by the probabilistic algorithm developed in the previous section and it represents a novelty in the job-search literature.
The network consists of 12 individuals centered on the $i-th$ worker; six on the left and the remaining six on the right.

$$\underbrace{\overbrace{(i-th)-6;(i-th)-5;........}^{Left}(i-th)\overbrace{........(i-th)+5;(i-th)+6}^{Right}}_{Social\ Network}$$

The effect of the local spillover is summarized in figure 6; this shows the adjustment path without local variables (the dashed line), the path arising by assuming $p=0.5$ (the lowest one) and the one resulting by assuming $p=0.1$ (the highest one)[18]. In other words, we have tried to compare the benchmark solution with a two opposite situations: in the first one, the local term is rather strong since the probability $p=0.5$ is relatively high. This corresponds to an individual for whom the social network is important for the search mechanism. On the contrary, when $p=0.1$ the network is weak and this induces a negative effect on matching efficiency, as the figure shows.
Whether the social networks increases or not the match efficiency depends on parameters; when p is sufficiently high, the local network increases individual probability to be hired, employment increases hence n grows making the local component more powerful. This virtuoso circle increases matching efficiency respect to the case without spillovers if the network component has a sufficient magnitude depending on Λ. But when p is not sufficient to start the positive feedback, it works in a reverse way, reducing n. The final result is that, for suitable parameters Γ and p, the social network reduces matching efficiency both along the transitional path and the steady state equilibrium.

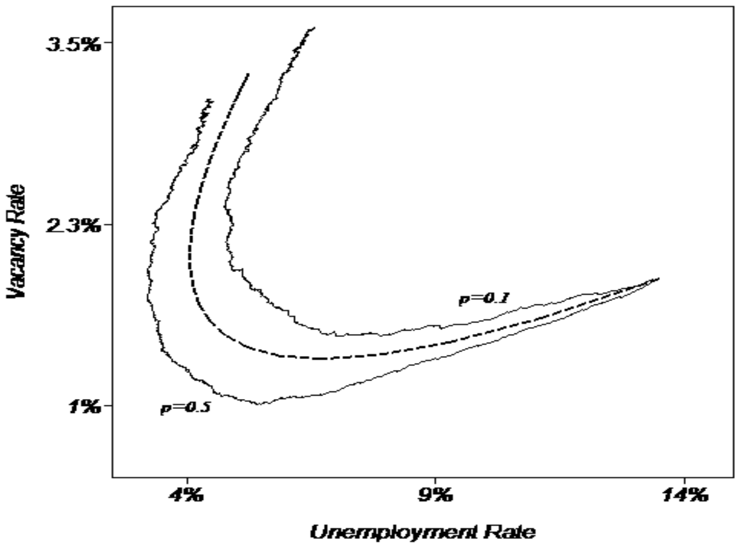

Figure 6

[17]These authors assume that each worker is re-matched at random with n workers each time a job offer arrives.

[18] As far as Γ is concerned, this has been held constant to 0.2 in order to put more weight on the local term.

This result depends on the endogeneity of the social network. This is made clearer by comparing the distribution of the network at the steady state when $p = 0.5$ and $p = 0.1$; results are in figure 7. It is rather clear that the distribution when *p=0.5* is more concentrated in the right part, at meaning the highest weight that the network has in this situation

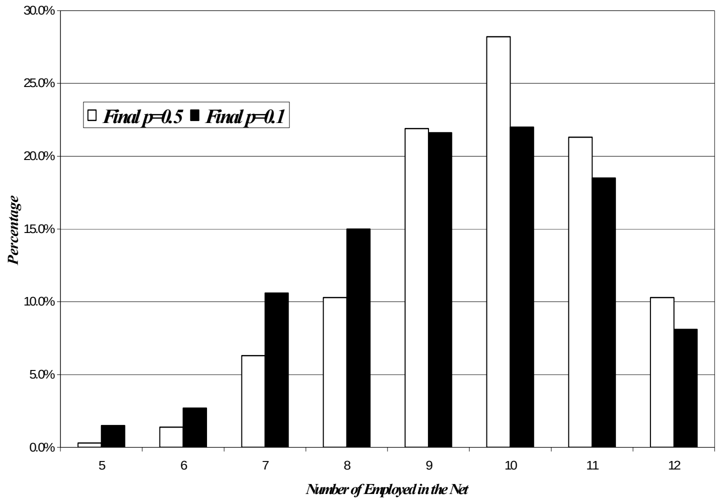

Figure 7

We are going to analyze further this point by assessing alternative ways of showing results in different conditions, i.e. when parameters change, by analyzing the dynamics of individuals over time. This provides us with a way of investigating social mobility hence the effects that the social network has on it, especially by looking at the possibility of endogenous traps of low mobility.
We start by showing a sort of transition matrix; table 2 sums up the percentage of occupancy time, for individuals starting from a given initial status, over the transition path to the steady state (18000 steps) and averaging over 20 replications. The table refers to the case of absence of social network and it must be read by rows; as an example, workers who started in the unemployed status 1 spend on average 10.55 % of the time necessary to achieve the steady state (which we can conventionally use to measure their life length) as unemployed , 82.32% as employed and so on.

	1	*2*	*3*
1	10.55	82.32	7.13
2	2.81	88.18	9.01
3	9.28	63.87	26.85

Table 2

This first measure of social mobility shows clearly the dependence on initial conditions not only for the economy as a whole, as stressed in the previous section, but on individuals life as well; individuals starting from employed have the higher occupancy time for the status of employed; it is worth noting the lower performance of individuals starting from out of labor force; this underlines the particular role played by this group of individuals that seems particularly disadvantaged. Even in absence of local effects, the dependence on initial conditions introduce a form of "club" convergence in the job market.
Now we compare the baseline social mobility matrix with the one arising from assuming social

network with $\Lambda = 0.8$, i.e. putting a relevant weight on the externality component, and with $p = 0.1$ (table 3); the simulations refers to figure 6. In other words we are assessing the trade off between the role played by the net and the low power of the net itself. The low value for p jointly with the high weight on the net component work as expected by increasing, although in a modest way, the average occupancy time for unemployed individuals, particularly for agents starting in the employed status (3.5 vs 2.8 of table 2). According to results previously shown in figure 6, the low probability of being hired jointly to the sensible weight of the externality component in driving transition from unemployed to employed reduces the matching efficiency. The magnitude of this negative effect is obviously positively correlated to the Λ coefficient: the higher the weight attached to the external component, the lower the matching efficiency if the coefficient p is small.

	1	*2*	*3*
1	10.95	81.31	7.74
2	3.50	87.10	9.45
3	10.11	66.30	23.58

Table 3

Conversely, when we let p to grow, we obtain a beneficial effect on employment; table 4 shows the previous simulations carried out with $p = 0.5$. In this case the net works in sustaining the matching efficiency, according to results shown in figure 6.

	1	*2*	*3*
1	9.91	82.71	7.38
2	2.53	88.77	8.70
3	8.41	63.69	27.90

Table 4

Another way at assessing the effects of social network on individual life-cycle is the comparison between average occupancy time in each status between the simulation obtained in absence of the net and the one obtained with it. Picture 7 shows in abscissa the average time of occupancy, in percent, of the whole population (1000 individuals) for status 1 in the benchmark simulation and in the vertical one the occupancy when the net is at work, with $p = 0.1$.

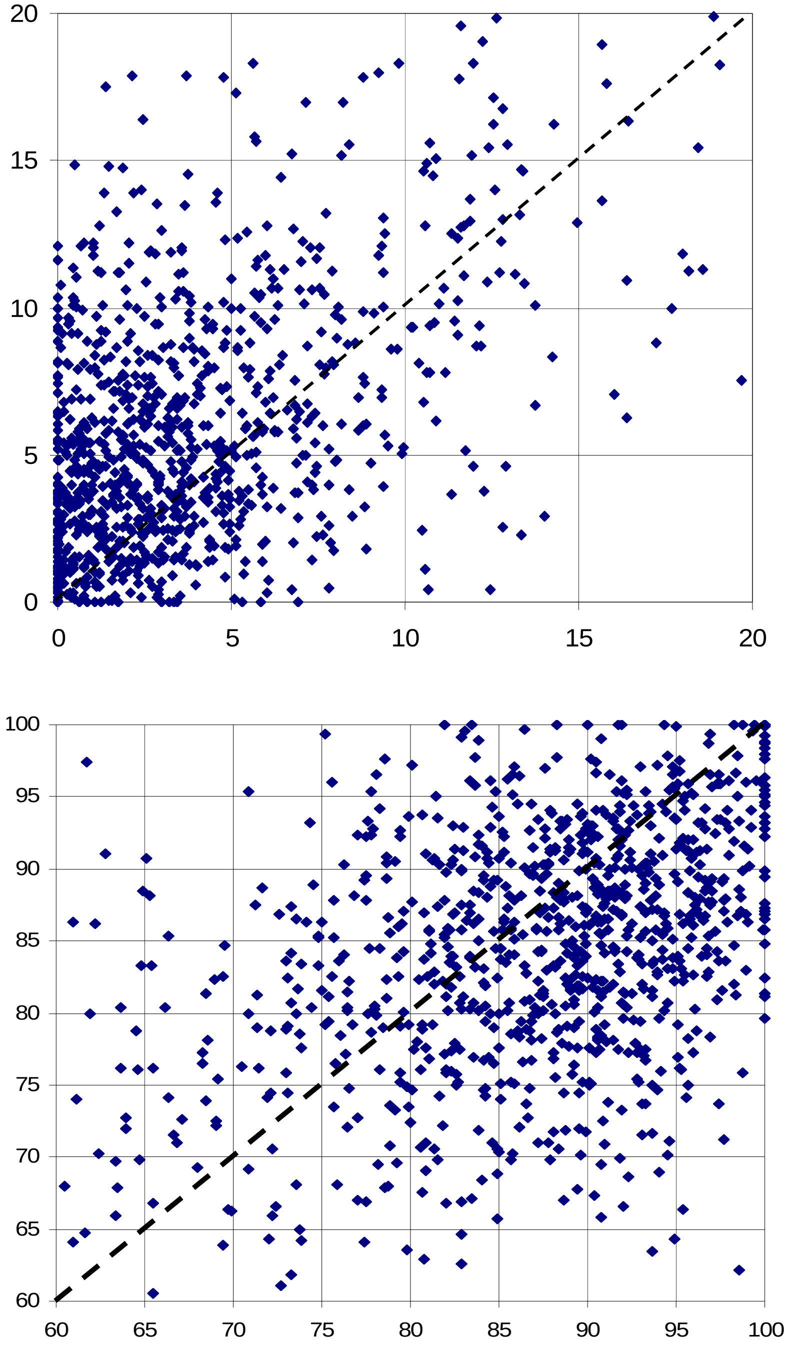

Figure 7

It is rather clear that the scatter is shifted up over the 45 degree line, at confirming that in the net simulation the unemployment status is, on average, more visited than the benchmark case. Conversely, the second graph refers to state 2 whose scatter shows a downward shifting.

Table 5 tries to investigate upon unemployment traps induced by the heterogeneity of transition rates due to the social network. The first row sums up the average time of occupancy for individuals starting from status one and having the lowest number of employed in the network in the initial condition (4 out of 12) in the benchmark simulation, while the second one shows the simulation with $p = 0.1$ of the same individuals and the third row with $p = 0.7$.

	1	*2*	*3*
1	11.26	85.18	3.56
2	14.28	79.58	6.13
3	6.42	89.52	4.06

Table 5

It is rather evident that the network can generates unemployment traps hence social discrimination when p is low; the average occupancy time of individuals starting in the unemployed situation in the second row is remarkably increased w.r.t. the benchmark simulation. Nevertheless, when p is sufficiently high the network reduces social discrimination.
Further numerical simulation here not reported for brevity, show that there is a trade off between the Λ and p parameters, as expected. In general the lower Λ, i.e. the weight of social network, the higher p in order to have beneficial effects on the reduction of unemployment spells and viceversa.
We can imagine other ways at modelling local effects in the transition rate but without adding a real improvement to results. As illustrative example, we can insert an externality directly in the matching function instead of using a linear combination as previously done. It is not hard to assume that the optimal search intensity could depend on local worker characteristics, such as sex, parental background, socioeconomic context and so on. Following our previous line of reasoning, we can make the optimal search intensity depending, in a positive way, by the number of employed workers in the surrounding of the individual. In other words, instead of providing a direct link between the local crowding component and the individual transition rate, we use a more endogenous channel, by letting the optimal search intensity be influenced by the spillover. This can be obtained in a twofold way: either by making β, the separation rate, depending negatively on the number of employed in the net, in the idea that separations are less frequent when there is a "good" state of the local economy or by reducing the search cost with the same effect, following the reasoning that an unemployed worker has a higher incentive to looking for a job when the state of the local component is favorable. The simulation we are going to show follows the former way, basically for algebraical tractability.
This time let p be the instantaneous probability that an employed worker does not separate when there is an employed in the neighbor and so $(1-p)^n$ is the probability, per unit of time, for a worker to become separated if she has n employed neighbors; $(1-(1-p)^n)$ is then the probability, per unit of time, for an employed not to be fired if she has n working neighbors. We modify the optimal search intensity of equation alphaemployed by making β a negative function of the term $(1-(1-p)^n)$; in this way the separation rate is decreasing when p and n increase for u and v given. In order to compare simulations with and without the local network, we are going to assume $\beta = 0.04[1-(1-(1-p)^n)] = 0.04(1-p)^n$, in this way β ranges between 0.04, the calibrated parameter in absence of social network, i.e. when $p = 0$, to zero when $p = 1$; moreover β is decreasing in n. Unlike the previous analysis, the social network can only increase matching efficiency, as β reduces in either case when p is positive w.r.t. the benchmark and, being the

optimal search intensity monotonically decreasing in β, this produces beneficial effects in the individual transition rate from U to E .
The functioning of this type of social net is as expected: an increase in p produces an improving in the matching efficiency. Figure 8 shows the usual deterministic benchmark simulation and the one arising with p=0.1. It is worth noting that, despite the low magnitude of p, the final effect on the stochastic simulation is quite remarkable, as the transition path shifts inwards bringing the Beveridge curve, the equilibrium value for u, v, well below the steady state obtained in absence of the social network.

Figure 8

Concluding, when we account for endogeneity of the social network composition, it is not so clear whether local effects are undoubtedly positive for the matching mechanism; as shown, this depends on the way the social network affects the matching function. Figure 6 refers to a more general type of social network, presenting a twofold type of dynamics, whilst figure 8 accounts for a more particular way of affecting the functioning of the matching process. Obviously alternative channels of influence can be analyzed but this does not change the final result; social networks affect remarkably the search and match process but in a way that are not well clear in advance. This section have shown that, by accounting for heterogeneity of transition rates over time and across agents due to the endogeneity of social network, the latter is able to produce either result, spanning from a positive to a negative effect on the equilibrium value. This type of analysis is neglected in the current literature and it can be performed only by suitable numerical schemes, as shown in this section.

Conclusions

This first attempt at modelling job search at individual level has interesting potentialities that the macro-dynamics can not capture. The debate on job search and determinants of the matching probabilities can only be faced by analyzing the single worker history; in particular, neighborhood effects can have important effects on the dynamics of the job market, both for the steady state and the transitional paths. The approach we are developing fits particularly well in coping with these spillover effects. Individual transition rates can depend on the social and economic neighbor of the individual and in general there is heterogeneity in such rates; this brings to equilibria depending on parameters and initial conditions and to dynamical path which are quite different by the ones we can describe by the aggregate equations.

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