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A stable or smoothly growing level of aggregate economic output would be most desireable for a prospering society. It would allow consumers, investors, government, and public agencies to make reliable plans. Yet, since centuries the actual behaviour of aggregate output has in most economies been characterized by oscillatory fluctuations, "booms" and "recessions". With a focus on the effects of interactions in production and deliveries, we investigate the behaviour of the flow of goods and services between the sectors of national economies. We use a dynamic variant of Leontief's classical input-output model[1,2] to represent the production activities, taking into account delays in the adaptation of price levels and production rates to a varying demand. On the basis of empirical data from the OECD, we compare analytical and numerical model implications with the behaviour of aggregate economic output over time. Our results indicate that the production rates and inventories may oscillate merely due to the network structure of commodity flows and the interactive adaptation within the economic sectors.

From the 19th century onwards, economics has been engaged in developing a variety of business cycle theories.[3,4] Most of them focus on dynamic effects between macroeconomic, i.e. aggregate variables. Because of insufficient adjustments in interest rates, wage rates and prices, investment, consumption expenditure, government spending, trade balance, money supply, etc. are conjectured to oscillate instead of reaching a stable value. According to the more recent 'real business cycle theory',[5,6] cyclical fluctuations in aggregate economic output are caused by unanticipated, *exogeneous* shocks which affect the economy and trigger more or less delayed adjustments in the aggregate variables. Yet, if it were not for an incessant series of shocks from outside, the repercussions of each single shock would eventually fade out.

The enormous complexity of the individual economic interactions from which aggregate output flows emerge, however, makes it difficult to trace back the causes of oscillatory variations to what happens at a less aggregate level. Only recently attempts have been made to explicitly account for a dynamic aggregation, many of them inspired by multi-particle systems in physics.[7−15] Currently, the subject of business cycles attracts new interest due to the progress in econophysics[15−17] and the theory of networks.[18−22] Our dis-aggregated model suggests *endogeneous* reasons for oscillations in the macroeconomic output flows: As a result of the economy's network structure, oscillations triggered by an exogeneous shock do not necessarily fade out in the absence of further shocks. Accordingly, cyclical oscillations appear as an inherent feature of decentralized decision making interconnected by the network structure of markets.

Like in Leontief's classical input-output model[1] we assume that each of m economic sectors $i \in \{1, ..., m\}$ produces one specific commodity. Let $Q_i(t) \geq 0$ denote the output of commodity i at time t produced per unit time Δt. The share $X_{ij}(t) \geq 0$ of that output is delivered as an input to the current production in sector j, and a share $Y_i(t) \geq 0$ is absorbed by final consumption. If $\sum_j X_{ij}(t) + Y_i(t)$ does not add up to $Q_i(t)$, this is assumed to result in a corresponding change $[N_i(t+\Delta t) - N_i(t)]$ of inventory $N_i(t)$ per unit time Δt. This assumption implies a *conservation equation* for commodities.[14] In the limit $\Delta t \to 0$ it reads

$$\frac{dN_i(t)}{dt} = Q_i(t) - \sum_{j=1}^{m} a_{ij} Q_j(t) - Y_i(t)\,, \tag{1}$$

considering that the commodity flow $X_{ij}(t)$ from sector i to sector j represents the share $a_{ij}Q_j(t) \geq 0$ of the entire inputs used by sector j. The input coefficients $a_{ij} \leq 1$ define an input matrix $\mathbf{A} = (a_{ij})$ reflecting the economy's technologically determined supply network and can be constructed from empirical data.[23] For simplicity, we will treat the input coefficients as constants.

The share of final consumption in sector i is assumed to show random fluctuations $\xi_i(t)$ over time around a certain average demand Y_i^0 and to decrease with an increase in the price $P_i(t)$ according to a standard demand function f_i:

$$Y_i(t) = [Y_i^0 + \xi_i(t)] f_i(P_i(t))\,. \tag{2}$$

Accordingly, the derivative $f_i'(P_i) = df_i(P_i)/dP_i$ is negative and the average value of $f_i(P_i(t))$ is normalised to one.

In our model, the prices $P_i(t)$ for the commodities in the different sectors are adjusted by the producers in response to two criteria: First, if the current inventory $N_i(t)$ exceeds some desired level N_i^0, prices tend to be cut and vice versa. A certain inventory level is desireable to cope with known variations in the order rate, possible breakdowns in production, etc. Second, if inventories are growing ($dN_i/dt > 0$), i.e. if the current production $Q_i(t)$ exceeds the current demand $Y_i(t) + \sum_j a_{ij}Q_j(t)$, this is an independent motive for cutting prices. The following equation for the relative price change expresses these responses and guarantees non-negative prices:

$$\frac{1}{P_i(t)}\frac{dP_i}{dt} = \nu_i \left(\frac{N_i^0}{N_i(t)} - 1 \right) - \frac{\mu_i}{N_i(t)}\frac{dN_i}{dt}\,. \tag{3}$$

Here, ν_i is an adaptation rate describing the sensitivity to deviations of the actual inventory $N_i(t)$ from the desired one N_i^0. μ_i is a dimensionless parameter reflecting the responsiveness to relative deviations $(dN_i/dt)/N_i(t)$ from the stationary equilibrium state.

Apart from price adjustments, undesired inventory levels and inventory changes can also be compensated for by adjusting current production. If the same criteria are applied, we find

$$\frac{1}{Q_i(t)}\frac{dQ_i}{dt} = \alpha_i\nu_i \left(\frac{N_i^0}{N_i(t)} - 1 \right) - \frac{\alpha_i\mu_i}{N_i(t)}\frac{dN_i}{dt}\,. \tag{4}$$

Here, α_i is the ratio between the adjustment rate of the output flow and the adjustment rate of the price in sector i.

Despite their mathematical similarity, Eqs. (3) and (4) have a surprisingly different impact on the macroeconomic dynamics (see caption of Fig. 1). The time-dependent behaviour close to the stationary equilibrium (where $N_i(t) = N_i^0$, $Y_i(t) = Y_i^0$, $P_i(t) = P_i^0$, and $Q_i^0 - \sum_j a_{ij}Q_j^0 =$

Y_i^0) can be studied by means of a linear stability analysis.[24] The linearized equations for the deviations $n_i(t) = N_i(t) - N_i^0$, $p_i(t) = P_i(t) - P_i^0$, and $q_i(t) = Q_i(t) - Q_i^0$ read:

$$\frac{dn_i}{dt} = q_i - \sum_j a_{ij} q_j - Y_i^0 f_i'(P_i^0) p_i - \xi_i(t)\,, \tag{5}$$

$$\frac{dp_i}{dt} = \frac{P_i^0}{N_i^0}\left(-\nu_i n_i - \mu_i \frac{dn_i}{dt}\right)\,, \tag{6}$$

$$\frac{dq_i}{dt} = \frac{\alpha_i Q_i^0}{N_i^0}\left(-\nu_i n_i - \mu_i \frac{dn_i}{dt}\right)\,. \tag{7}$$

This system of coupled differential equations describes the response of the inventories, prices, and production rates to variations $\xi_i(t)$ in the demand. Denoting the m eigenvalues of the input matrix $\mathbf{A}$ by ω_i with $|\omega_i| < 1$, the $3m$ eigenvalues of the linearized equations are 0 (m times) and

$$\lambda_{i,\pm} = \frac{1}{2}\left(- A_i \pm \sqrt{(A_i)^2 - 4B_i}\right) \tag{8}$$

with $A_i = \mu_i[C_i + \alpha_i D_i(1-\omega_i)]$, $B_i = \nu_i[C_i + \alpha_i D_i(1-\omega_i)]$, $C_i = P_i^0 Y_i^0 |f_i'(P_i^0)|/N_i^0$ and $D_i = Q_i^0/N_i^0$. The exact validity of formula (8) requires either the matrix $\mathbf{A}$ to be diagonal or $\mu_i C_i$, $\alpha_i \mu_i D_i$, $\nu_i C_i$ and $\alpha_i \nu_i D_i$ to be sector-independent constants (otherwise the eigenvalues must be numerically determined).

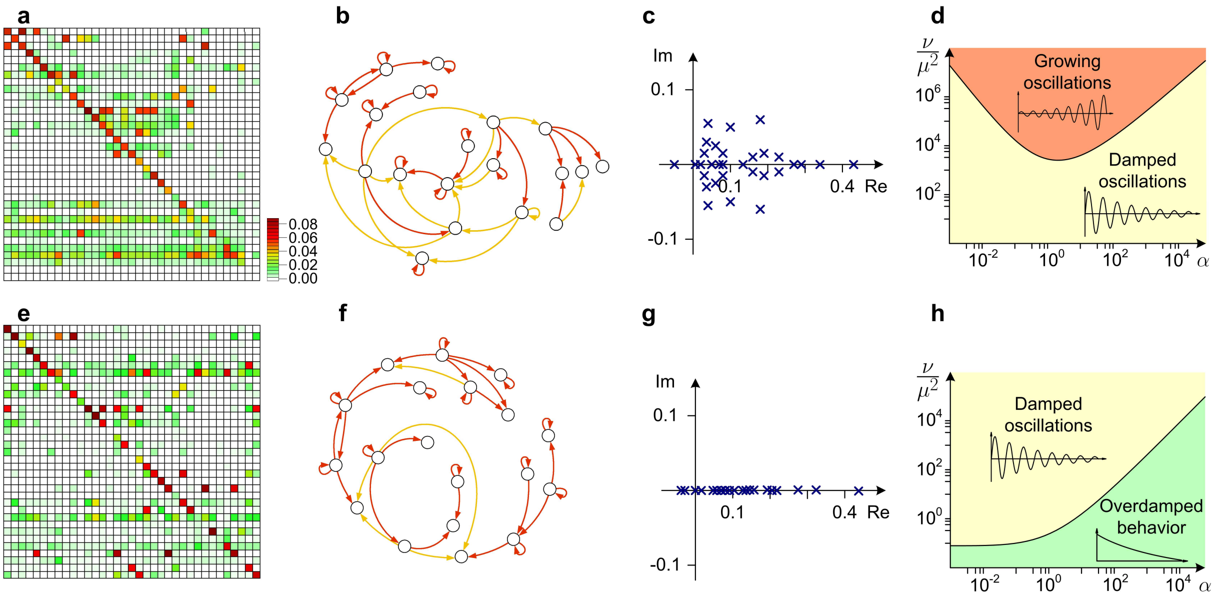

FIG. 1. Properties of our dynamic model of dis-aggregate output flows for a characteristic input matrix specified as average input matrix of several countries (top) and for a synthetic input matrix generated by random changes of input matrix entries until the number of complex eigenvalues was eventually reduced to zero (bottom). Subfigures **(a), (e)** illustrate the color-coded input matrices $\mathbf{A}$, **(b), (f)** the corresponding network structures, when only the strongest links (commodity flows) are shown, **(c), (g)** the eigenvalues $\omega_i = \text{Re}(\omega_i) + \text{iIm}(\omega_i)$ of the respective input matrix $\mathbf{A}$, and **(d), (h)** the phase diagrams indicating the stability behaviour of the model equations (1) to (4) on a double-logarithmic scale as a function of the model parameters $\alpha_i = \alpha$ and $\nu_i/{\mu_i}^2 = \nu/\mu^2 = V/M^2$. The other model parameters were set to $\nu_i = C_i = D_i = P_i^0 = N_i^0 = Y_i^0 = 1$. The behaviour in the limit of small and large parameter values can be understood analytically: (i) If $(A_i)^2/B_i \gg 1$, the eigenvalues become $\lambda_{i,-} \approx -A_i$ and $\lambda_{i,+} \approx -B_i/A_i = -\nu_i/\mu_i$. This corresponds to a relaxation to the equilibrium state in the case of a large responsiveness $\mu_i \gg 1$. An overdamped behaviour is found if all eigenvalues ω_i are real numbers or if all $\alpha_i = 0$, otherwise one expects network-induced oscillations. Interestingly enough, $\mu_i \gg 1$ implies $(dP_i/dt)/(\mu_i P_i) \approx 0$, so that Eq. (3) reduces to $dN_i/dt \approx \nu_i[N_i^0 - N_i(t)]/\mu_i$. Therefore, $N_i(t) \approx N_i^0$ and $dQ_i/dt \approx 0$ (i.e. $Q_i \approx Q_i^0$). Inserting this into Eq. (1) yields an implicit equation for the price $P_i(t)$ as a function of the fluctuations $\xi_i(t)$ in the consumption rate, as usually assumed in economics. It reads $[Y_i^0 + \xi_i(t)]f_i(P_i(t)) \approx Q_i^0 - \sum_j a_{ij}Q_j^0 = \text{const.}$ (ii) In the case $\alpha_i \to 0$ of fast price adjustment, the corresponding eigenvalues $\lambda_{i,\pm}$ are given by $2\lambda_{i,\pm} = -\mu_i C_i \pm \sqrt{(\mu_i C_i)^2 - 4\nu_i C_i}$, independently of $\mathbf{A}$. That is, we expect an exponential relaxation to the economic equilibrium for $0 < \nu_i/{\mu_i}^2 < C_i/4$, but otherwise damped oscillations. (iii) In the case $\alpha_i \gg 1$ of slow price adjustment, one can eliminate Eq. (6) by assuming $p_i(t) \approx 0$ and $P_i(t) \approx P_i^0$, so that $dn_i/dt \approx q_i - \sum_j a_{ij}q_j - \xi_i$. Deriving this with respect to time and inserting Eq. (7) delivers $d^2n_i/dt^2 + \sum_j(\delta_{ij} - a_{ij})\,\alpha_j D_j[\mu_j dn_j/dt + \nu_j n_j(t)] \approx -d\xi_i/dt$, where $\delta_{ij} = 1$ for $i = j$, otherwise $\delta_{ij} = 0$. If we assume sector-independent constants $\alpha_i\mu_i D_i = M$ and $\alpha_i\nu_i D_i = V$, the $2m$ eigenvalues $\lambda_{i,\pm}$ are given by $2\lambda_{i,\pm} = -M(1-\omega_i) \pm \sqrt{[M(1-\omega_i)]^2 - 4V(1-\omega_i)}$. Surprisingly, for empirical input matrixes $\mathbf{A}$, one never finds an overdamped, exponential relaxation to the economic equilibrium, but network-induced oscillations due to complex eigenvalues ω_i. For large values of α_i and $\nu_i/{\mu_i}^2$, see Eq. (9), the oscillations are growing (top). Instead, for input matrices $\mathbf{A}$ with real eigenvalues ω_i only, oscillations are damped for large values of $\nu_i/{\mu_i}^2$, while $\min_i({A_i}^2 - 4B_i) > 0$, i.e. small enough values $\nu_i/{\mu_i}^2 < \min_i[C_i + \alpha_i D_i(1-\omega_i)]/4$ can guarantee an overdamped behaviour (bottom).

Stable, overdamped system behaviour results, if the eigenvalues $\lambda_{i,\pm}$ are real and negative (see bottom of Fig. 1). This requires small values of $\nu_i/{\mu_i}^2$, i.e. a small sensitivity to deviations from the desired inventories N_i^0 compared to the responsiveness to a disequilibrium $dN_i/dt \neq 0$. If not all eigenvalues ω_i of the input matrix $\mathbf{A}$ are real numbers or all $\alpha_i = 0$ (which would require instantaneous price adjustments), the eigenvalues $\lambda_{i,\pm}$ are complex numbers. Interestingly enough, this is the typical case, as the network structure of economic output flows is characterised by complex eigenvalues $\omega_i = \text{Re}(\omega_i) + \text{i}\,\text{Im}(\omega_i)$. Therefore, *the generic behaviour is network-induced oscillations around the equilibrium state* (see top of Fig. 1 and Fig. 2). The origin of these oscillations may, but do not have to be exogeneous shocks. For large values of $\nu_i/{\mu_i}^2$, oscillatory variations in aggregate output may even arise as an emergent phenomenon and grow over time. However, because of the non-linearities in Eqs. (1) to (4) and phase shifts between the oscillations in different sectors, the oscillation amplitude of the aggregate output is limited (see Fig. 2).

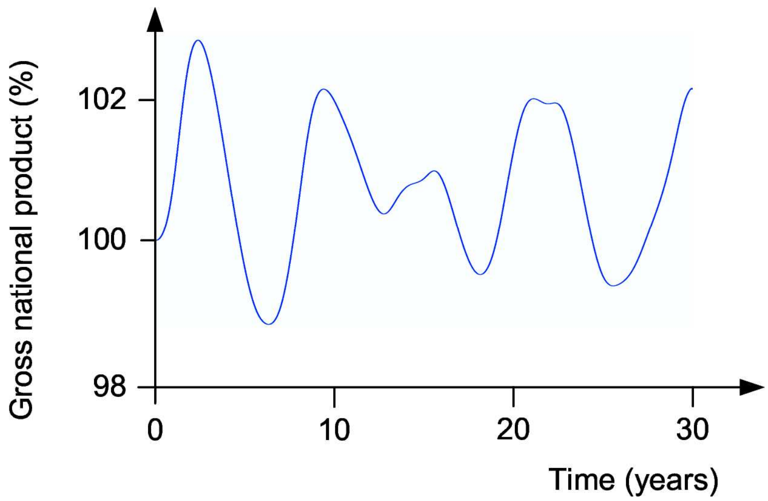

FIG. 2. Typical simulation result of the time-dependent gross domestic product $\sum_i Q_i(t)P_i(t)$ in percent, i.e. relative to the initial value. The input matrix was chosen as in Fig. 1a–d, but Y_i^0 was determined from averaged input-output data. Q_i^0 was obtained from the equilibrium condition, and the fluctuations $\xi_i(t)$ were specified as a Gaussian white noise with mean value 0 and variance $\sigma = 10.000$ (about 10% of the average final consumption). The initial prices $P_i(0)$ were selected from the interval [0.9;1.1]. Moreover, in this example we have assumed $f_i(P_i) = \max[0, 1+d(P_i - P_i^0)]$ with $d = f'(P_i^0) = -10$ and the parameters $\nu_i = 0.1$, $\mu_i = 0.0001$, $\alpha_i = 1 = P_i^0$, and $N_i^0 = Y_i^0$. Although this implies a growth of small oscillations (cf. Fig. 1d), the oscillation amplitudes are rather limited. This is due to the non-linearity of model equations (1) to (4) and due to the phase shifts between oscillations of different economic sectors i. Note that irregular oscillations with frequencies between 4 to 6 years and amplitudes of about 2.5% are qualitatively compatible with empirical business cycles. The long-term growth of national economies was intentionally not included in our model in order to separate this effect from network-induced instability effects.

It is surprising that increasing oscillation amplitudes are found if the adaptation rates ν_i are large. Nevertheless, many common production strategies suggest to keep constant inventories N_i^0, which potentially destabilizes economic systems. Ideal values should lie below the instability line

$$\frac{\nu_i}{{\mu_i}^2} = \min_i \left[\Big\{ C_i + \alpha_i D_i [1 - \mathrm{Re}(\omega_i)] \Big\} \left(1 + \frac{\{C_i + \alpha_i D_i [1 - \mathrm{Re}(\omega_i)]\}^2}{[\alpha_i D_i \mathrm{Im}(\omega_i)]^2} \right) \right] \tag{9}$$

(see Fig. 1d), which can be derived from the condition $\max_i \mathrm{Re}(\lambda_{i,\pm}) = 0$. Note that the stabilizing adjustment to changes dN_i/dt in the inventories is a difficult task, as the time derivatives are fluctuating quantities. The use of exponentially smoothed data, however, would cause delayed reactions. This may be avoided by enhanced forecast methods. Network theory[18−22] can also make significant contributions to the construction of stable, robust, and adaptive supply networks. On the basis of Eq. (9) structural policies can be identified, which would have a stabilizing effect. An example of an optimized supply network showing stable behaviour is illustrated in Fig. 1e–h. Other examples are regular supply hierarchies (distribution networks) or supply ladders (see Fig. 10b, c in Ref. [14]). The results and techniques presented in this study may be also used to enhance the robustness of production processes, to model networks in biology such as metabolic networks, and to optimize disaster mangement.[25]

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