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Industry and Labour Dynamics II

Proceedings of the wild@ace 2004 conference

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CLUSTER-BASED APPROACH TO THE AGENT-BASED MODELS (AMB)

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Abstract

The non-stationary cluster size distribution in a network of agents which evolves without preferential behavior has been studied at early and late stages of clusters formation, when they are expected to grow and decay mainly by gaining and losing single agents. We developed a further model for the dynamics of an open system formed by time-dependent number of agents. Groups of agents (clusters) change their sizes by all possible transitions: addition/removal of agent(s) (monomers, dimers etc.) to/from the system, coagulation of two clusters of sizes r and $k-r$, where $1 < r < k-1$ ($k > 3$), attachment of monomers, dimers, trimers etc. to clusters of size k or disappearance of a cluster of size k by coagulation with another cluster, and detachment of monomers, dimers, trimers etc. Both non-zero initial cluster size distribution and when only free agents (monomers) are present in the system at the initial moment have been studied, and we describe accordingly the effect of arbitrary pre-existing clusters on the time evolution of the interacting agents.

PACS: 02.60.Cb; 05.40.-a; 89.65.Gh

Keywords: Agents-clusters; Fragmentation; Coagulation; Cluster size distribution;
Market structure

1. Introduction

Complex systems typically have the following characteristics. First, they consist of a large number of agents interacting with each other in many different ways. Secondly, interest focuses on collective properties of the system as a whole, which may result in emergent structures of great complexity at larger scales or after long times. Thirdly, random or stochastic effects are important: both determinism and chance are needed to describe the reality. Fourthly, the system may be adaptive, a property most often found in the biological and social sciences. For example, the stochasticity can lead to the production of “errors” (e.g. mutations) which may give rise to better variants and, through selection, eventually lead to the development of new complex structures in systems far from equilibrium. These properties are frequently found in the physical sciences, and are the domain of non-equilibrium statistical mechanics. Because of the real limitations in the use of analytical methods to study such problems, it is often necessary to resort to numerical methods, and the advent of computer simulations has led to an increase of scientific activity in this area that has emerged nowadays as a major subject of interdisciplinary research. In particular, many papers were recently published in the field of application of the methods of statistical physics, thermodynamics, synergetics, etc. in economics [1-15], and they are on the increase. It is believed, for example, that the universal aspects observed in the empirical analysis of finance fluctuations and economic organizations, i.e. the empirically observed power-law distributions, can arise from an interplay between random, stochastic growth and the complex structure of the system, and there are a number of kinetic models based on cluster (group of agents) idea which have been introduced recently to study such kind of fascinating behavior, that is the herding and group kinetics of agents in a market. In general, a *cluster* represents an aggregate formed by a number of similar connected elements-units or agents (individuals, enterprises, industries, countries) that are thought to be unitary and indivisible. To each agent it is possible to assign,

for example, some wealth-parameter that represents its welfare. The exact choice of this parameter is not straightforward. For instance, when thinking of countries in the world economy, the GDP, GNP or some function of macroeconomic indicators could be a reasonable choice. In the case of companies, equity, share price or some suitable combination of them with outstanding debt are reasonable candidates. Connected agents are agents sharing the same information and making the same decision between, for example, buying, selling or doing nothing. Therefore, a cluster is thought to be a group of agents that can exchange information, either directly or through intermediary connected agents, and do the same action on the market. We use the generic term *size* to mean any of economic variables such as sales, number of employees, assets, etc., because the previous reports [3, 4] have showed them to scale in an identical way.

A theoretical model based on the cluster theory was developed and used to simulate the dynamics of complex systems, composed of a number of interacting agents-clusters, with different sizes M [15]. The average group (cluster) size problem was also solved for different values of M , and the process of relaxation in the system was studied. In particular, the role of attachment probability was described by comparison between this model and other kinetic models of random growing networks and herding phenomena, and the size effect on the formation of the groups of agents, under the condition that only free agents are present in the system at the initial moment, was elucidated. Therefore, it is worth looking further into the role of pre-existing random generated clusters with different sizes in the non-stationary cluster size distribution, as well as into the more general case of an open system, and we present in this paper our findings relating to these problems too.

2. The model

2.1 The generic case of a closed system and no pre-existing clusters

In the framework of the cluster approach the following two basic assumptions, which allow a mathematical formalism to be developed for a detailed description of the evolution, are indispensable [15]:

- (i) There exist clusters in the initial state which consist of different number n of units ($n=1, 2, \dots$);
- (ii) Transformations of n -sized clusters into m -sized ones at time t occur with certain, in general, time-dependent frequencies (transition rates) $f_{nm}(t)(s^{-1})$ ($n, m=1, 2, \dots$).

The evolution of the process is sought to be described by function $Z_n(t)$, which represents the solution of the kinetic master equation and characterizes the time-dependence of the concentration of clusters of size n . Figure 1 shows schematically how n -sized cluster can increase or decrease its size. In particular, the arrow beginning from size n and ending at size m on the size axis symbolizes the quantity $f_{nm}(t)Z_n(t)$, which gives the number of $n \rightarrow m$ transitions undergone by the n -sized clusters per unit time, divided by the number of interacting agents-clusters $N(t)$, where $N(t) = \sum_{k=1}^M n_k(t)$, M is the number of agents in the system, and $n_k(t)$ is the number of groups (clusters) of size k at time t . Then, the concentration of clusters of size n will be diminished per unit time by the quantity:

$$\sum_{m=1}^M f_{nm}(t)Z_n(t). \quad (1)$$

Conversely, the arrows ending at size n illustrate the role of the reverse, i.e. the $m \rightarrow n$, transitions: owing to them $Z_n(t)$ will increase per unit time by the quantity:

$$\sum_{m=1}^M f_{mn}(t)Z_m(t). \quad (2)$$

On the other hand, the change of $Z_n(t)$ per unit time is expressed mathematically by the derivative $dZ_n(t)/dt$. The balance between the above quantities thus leads to the sought master equation for a closed system:

$$\frac{dZ_n(t)}{dt} = \sum_{m=1}^M [f_{mn}(t)Z_m(t) - f_{nm}(t)Z_n(t)]. \quad (3)$$

Equation (3) is a set of ordinary differential equations of first order. In general, these equations are non-linear because of the dependence of the transition frequencies on the unknown cluster concentration $Z_n(t)$. Clearly, $Z_n(t)$ and M are connected by the relation

$$N(t) \sum_{n=1}^M nZ_n(t) = M, \text{ and the initial cluster size distribution } Z_n(0) \text{ is considered to be } a \text{ priori}$$

known.

We can now easily introduce the so-called Szilard model for the processes of type $[Z_1]+[Z_n] \leftrightarrow [Z_{n+1}]$ with time-independent transition frequencies, occurring especially at the early stage when it is unlikely for the clusters of $n=2, 3, \dots$ units to interact, because their concentration is still rather low. This is illustrated in Figure 2 in which the arrows symbolize the number of forward ($n \rightarrow n+1$) and backward ($n \rightarrow n-1$) transitions. Denoting $f_n = f_{n,n+1}(t)$, $f_{n-1} = f_{n-1,n}(t)$, $g_n = f_{n,n-1}(t)$, $g_{n+1} = f_{n+1,n}(t)$, where $f_{nm}(t) = 0$ for $|n-m| > 1$ and $f_{nm}(t) \neq 0$ only for $|n-m| = 1$, the clusters will change size by nearest-size transitions and the equation (3) becomes:

$$\frac{dZ_n(t)}{dt} = f_{n-1}Z_{n-1}(t) - g_nZ_n(t) - f_nZ_n(t) + g_{n+1}Z_{n+1}(t). \quad (4)$$

By definition, $f_0=0$, $g_1=0$ and $Z_{M+1}=0$. The initial condition for a closed system ($M=const$), when at $t=0$ there are only agents in the system with concentration Z_1 , is $Z_1(0)=1$, and $Z_n(0)=0$ ($n=2, 3, \dots, M$). Meanwhile, we can apply two different sets of conditions for the cluster size distribution for $n=1$ and $n=M$. The first one, in conformity with the assumption that the actual concentration of agents in a system in partial equilibrium tends to the equilibrium one, is $Z_1(t) \approx 1$ and $Z_M(t)=0$. In this case we shall require $Z_1(t)$ to remain constant with time, and this

assumption is based on the facts that the initial “consumption” of agents by the growing clusters is negligible, and the concentration of agents, Z_1 , is thought to be much larger than $Z_n(t)$ for $n=2, 3, \dots, M-1$ and remains thus nearly unchanged. So the equation (4) becomes a set of $M-2$ ordinary linear differential equations of first order in the $M-2$ unknowns $Z_2(t), Z_3(t), \dots, Z_{M-1}(t)$. The second set is time-dependent $Z_1(t)$ and $Z_M(t)=0$, and this leads us to a group of $M-1$ homogeneous ordinary linear differential equations which can be solved straightforward as a particular way of the first one. For that reason, we look below just into more complex initial case of constant boundary conditions.

The first step in solving equation (4) is to homogenize it by presenting $Z_n(t)$ in form of the sum of stationary cluster size distribution, X_n , and unknown deviation of $Z_n(t)$ from X_n , $y_n(t)$:

$$Z_n(t) = X_n + y_n(t), \quad (5)$$

where $y_n(t)$ satisfies the initial condition $y_n(0) = -X_n$, and $dy_n(t)/dt = f_{n-1}y_{n-1}(t) - (f_n + g_n)y_n(t) + g_{n+1}y_{n+1}(t)$ ($n=2, 3, \dots, M-1$) which is a set of $M-2$ already homogeneous ordinary linear differential equations of first order with time-independent coefficients. For any fixed $n=2, 3, \dots, M-1$ we shall have $M-2$ linearly independent particular solutions $y_{ni}(t)$ of the form:

$$y_{ni}(t) = a_{ni} \exp(-\lambda_i t) \quad (i=2, 3, \dots, M-1), \quad (6)$$

where a_{ni} are constants (to ensure the linear independence of $y_{ni}(t)$ we shall require that not all a_{ni} equal zero simultaneously), and $\lambda_i > 0$ is the i th eigenvalue, i.e. the i th root of the characteristic equation

$$\begin{vmatrix} f_2 + g_2 - \lambda & -g_3 & 0 & \cdots & 0 & 0 & 0 \\ -f_2 & f_3 + g_3 - \lambda & -g_4 & \cdots & 0 & 0 & 0 \\ 0 & -f_3 & f_4 + g_4 - \lambda & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & f_{M-3} + g_{M-3} - \lambda & -g_{M-2} & 0 \\ 0 & 0 & 0 & \cdots & -f_{M-3} & f_{M-2} + g_{M-2} - \lambda & -g_{M-1} \\ 0 & 0 & 0 & \cdots & 0 & -f_{M-2} & f_{M-1} + g_{M-1} - \lambda \end{vmatrix} = 0$$

The above determinant represents a polynomial of degree $M-2$ which has $M-2$ simple roots $\lambda_2, \lambda_3, \dots, \lambda_{M-1}$. The next step is, therefore, to find these roots, and then to determine, for each $i=2, 3, \dots, M-1$, the constants a_{ni} with the help of the recursion formulae:

$$\begin{aligned} (f_2+g_2-\lambda_i)a_{2i}-g_3a_{3i}&=0, \\ -f_{n-1}a_{n-1,i}+(f_n+g_n-\lambda_i)a_{ni}-g_{n+1}a_{n+1,i}&=0, \quad (n=3,4, \dots, M-2) \\ -f_{M-2}a_{M-2,i}+(f_{M-1}+g_{M-1}-\lambda_i)a_{M-1,i}&=0, \end{aligned} \quad (7)$$

where it is convenient to set, without loss of generality, $a_{M-1,i}=1$ for each $i=2, 3, \dots, M-1$.

We can now use the $M-2$ linearly independent solutions $y_{ni}(t)$ from (6) in order to represent the general solution $y_n(t)$ as a linear combination of them:

$$y_n(t) = \sum_{i=2}^{M-1} c_i a_{ni} \exp(-\lambda_i t) \quad (n=2, 3, \dots, M-1). \quad (8)$$

The last step is to find the $M-2$ unknown constants c_i which are the solution of the linear algebraic set of $M-2$ equations resulting from using the initial condition for y_n in (8):

$$\sum_{i=2}^{M-1} c_i a_{ni} = -X_n \quad (n=2, 3, \dots, M-1). \quad (9)$$

In accordance with the Cramer rule, c_i is given by $c_i=d_i/d'$ ($i=2, 3, \dots, M-1$), where d_i and d' are the following determinants of order $M-2$:

$$d_i = \begin{vmatrix} a_{22} & a_{23} & \cdots & a_{2,i-1} & -X_2 & \cdots & a_{2,M-1} \\ a_{32} & a_{33} & \cdots & a_{3,i-1} & -X_3 & \cdots & a_{3,M-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{M-1,2} & a_{M-1,3} & \cdots & a_{M-1,i-1} & -X_{M-1} & \cdots & a_{M-1,M-1} \end{vmatrix}, \quad (10)$$

$$d' = \begin{vmatrix} a_{22} & a_{23} & \cdots & a_{2,i-1} & a_{2i} & \cdots & a_{2,M-1} \\ a_{32} & a_{33} & \cdots & a_{3,i-1} & a_{3i} & \cdots & a_{3,M-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{M-1,2} & a_{M-1,3} & \cdots & a_{M-1,i-1} & a_{M-1,i} & \cdots & a_{M-1,M-1} \end{vmatrix}. \quad (11)$$

Using (9)-(11) and inserting $y_n(t)$ from (8) into (5), we find the solution of the problem:

$$Z_n(t) = X_n + \sum_{i=2}^{M-1} (d_i / d') a_{ni} \exp(-\lambda_i t) \quad (12)$$

or, equivalently,

$$Z_n(t) = X_n \left\{ 1 - \left[\sum_{i=2}^{M-1} d_i a_{ni} \right]^{-1} \sum_{i=2}^{M-1} d_i a_{ni} \exp(-\lambda_i t) \right\}, \quad (13)$$

where $X_n = \frac{f_1 f_2 \dots f_{n-1}}{g_2 g_3 \dots g_n} \left[1 + \sum_{m=2}^{M-1} \frac{g_2 g_3 \dots g_m}{f_2 f_3 \dots f_m} \right]^{-1} \sum_{m=n}^{M-1} \frac{g_2 g_3 \dots g_m}{f_2 f_3 \dots f_m}$ ($n=2, 3, \dots, M-1$)

is the stationary cluster size distribution. Equations (12) or (13) represent the time-dependent cluster size distribution which has an origin at no preexisting clusters in the system. We note that, as it should be, $Z_n(t) \rightarrow X_n$ for $t \rightarrow \infty$. Meanwhile, the exact expressions for the roots λ_i can be found analytically only for $M-2 \leq 4$, i.e. for $M \leq 6$. For $M > 6$ we must resort to numerical methods for solving the problem. The solution in case of the second set of conditions, i.e. time-dependent $Z_1(t)$ and $Z_M(t) = 0$, is just the non-stationary sum of equation (12) starting from $i=1$.

2.2 The case of arbitrary pre-existing clusters

The evolution of the process is sought to be described by the function $Z_n(t)$, which represents the solution of the kinetic master equation (4) and characterizes the time-dependence of the concentration of clusters of size n , where the initial cluster size distribution $Z_n(0)$ is considered to be *a priori* known. Note that one time step in this formulation corresponds to one update in the numerical simulation under time-dependent boundary conditions $Z_1(t) \neq 0$ and $Z_M(t) = 0$, and non-zero random initial cluster size distribution $Z_n(0) \neq 0$ ($n=1, 2, 3, \dots, M-1$). So the equation (4) becomes a set of $M-1$ homogeneous ordinary linear differential equations of first order in the $M-1$ unknowns $Z_1(t), Z_2(t), Z_3(t), \dots, Z_{M-1}(t)$. In this case the problem can be solved in a straightforward manner, and the solution $Z_n(t)$ takes a new

form without the stationary cluster size distribution part, and $i=1, 2, \dots, M-1$. The d_i is the only parameter affected by the non-zero initial cluster size distribution, and now it is given by the following determinant of order $M-1$:

$$d_i = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1,i-1} & Z_1(0) & \cdots & a_{1,M-1} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2,i-1} & Z_2(0) & \cdots & a_{2,M-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{M-1,1} & a_{M-1,2} & a_{M-1,3} & \cdots & a_{M-1,i-1} & Z_{M-1}(0) & \cdots & a_{M-1,M-1} \end{vmatrix}, \quad (14)$$

where a_{ni} are the above constants obtained with the help of transition frequencies f_n and g_n . For $M>5$ we must resort to numerical methods for solving the problem, because the exact analytical solution can be found only for $M\leq 5$.

2.3 The case of an open system with preferential attachment and fragmentation

The calculations are based on the following two basic assumptions: 1) There exist clusters in the initial state which consist of different number k of agents ($k=1, 2, \dots$); 2) Transformations of r -sized clusters into k -sized ones at time t occur with certain transition probability p_i ($i=1\div 4$), where $\sum_{i=1}^4 p_i = 1$. This is illustrated in Fig.3 in which the arrows symbolize all possible forward/backward transitions which take into account at each time step both addition with probability p_1 and removal with probability p_4 of agent(s) (monomers, dimers etc.) to (from) the system, coagulation with probability p_2 of two clusters of sizes r and $k-r$, where $1 < r < k-1$ ($k > 3$), **1**, attachment of dimers, trimers etc. to clusters of size k or disappearance of a cluster of size k by coagulation with another cluster, **2**, detachment with probability p_3 of dimers, trimers etc. from clusters of size $r > k+1$, **3**, and detachment of dimers, trimers etc. from clusters of size k , **4**. The evolution of the process is sought to be described by the function $n_k(t)$, the number of groups (clusters) of size k at time t , which represents the solution of the kinetic master equation:

$$k=1, \quad \frac{dn_1(t)}{dt} = p_1 - p_4 - \frac{p_2 n_1(t)}{M(t)} \sum_{r=1}^{M(t)-2} r n_r(t) + \frac{p_3}{M(t)} \sum_{r=2}^{M(t)-1} r(r-1) n_r(t) n_{r-1}(t) \quad (15.a)$$

$$k=2, 3, \dots, M(t)-2,$$

$$\begin{aligned} \frac{dn_k(t)}{dt} = & \frac{p_2}{M(t)} \left[\sum_{r=1}^{k-1} r(k-r) n_r(t) n_{k-r}(t) - k n_k(t) \sum_{r=1}^{M(t)-k-1} r n_r(t) \right] \\ & + \frac{p_3}{M(t)} \left[\sum_{k < r < M(t)} r(r-k) n_r(t) n_{r-k}(t) - k n_k(t) \sum_{r=1}^{k-1} r n_r(t) \right] \end{aligned} \quad (15.b)$$

$$k=M(t)-1,$$

$$\begin{aligned} \frac{dn_{M(t)-1}(t)}{dt} = & \frac{p_2}{M(t)} \sum_{r=1}^{M(t)-2} r(M(t)-r-1) n_r(t) n_{M(t)-r-1}(t) \\ & - \frac{p_3}{M(t)} (M(t)-1) n_{M(t)-1}(t) \sum_{r=1}^{M(t)-2} r n_r(t) \end{aligned} \quad (15.c)$$

The number of interacting clusters is $N(t) = \sum_{k=1}^{M(t)-1} n_k(t)$, and the number of agents in the open

system $M(t) = \sum_{k=1}^{M(t)-1} k n_k(t)$, where $n_{M(t)}(t) \equiv 0$. The evolution of $N(t)$ and $M(t)$ can be

represented accordingly by equations:

$$\frac{dN(t)}{dt} = \frac{d}{dt} \left(\sum_{k=1}^{M(t)-1} n_k(t) \right) = p_1 - p_4, \quad (16.a)$$

$$\begin{aligned} \frac{dM(t)}{dt} = & \frac{d}{dt} \left(\sum_{k=1}^{M(t)-1} k n_k(t) \right) = p_1 - p_4 + \frac{p_2}{M(t)} \sum_k \left[k [k n_k(t)]^2 + k n_k(t) \sum_{r=k+1}^{M(t)-k-1} r(k+r) n_r(t) \right] \\ & - \frac{p_3}{M(t)} \sum_{k=1}^{M(t)-2} k^2 n_k(t) \sum_{r=k+1}^{M(t)-1} r n_r(t) \end{aligned} \quad (16.b)$$

Without equivalent (unphysical) transitions the above rate equations (15) change to a new set:

$$k=1, \quad \frac{dn_1(t)}{dt} = p_1 - p_4 - \frac{p_2 n_1(t)}{M(t)} \sum_{r=1}^{M(t)-2} r n_r(t) + \frac{p_3}{M(t)} \sum_{r=2}^{M(t)-1} r(r-1) n_r(t) n_{r-1}(t) \quad (17.a)$$

$$k=2, 3, \dots, M(t)-2, \alpha_{r,k-r} = \begin{cases} 1, & r = k - r \\ 1/2, & r \neq k - r \end{cases}$$

$$\begin{aligned} \frac{dn_k(t)}{dt} = & \frac{p_2}{M(t)} \left[\sum_{r=1}^{k-1} \alpha_{r,k-r} r(k-r)n_r(t)n_{k-r}(t) - kn_k(t) \sum_{r=1}^{M(t)-k-1} rn_r(t) \right] \\ & + \frac{p_3}{M(t)} \left[\sum_{k < r < M(t)} r(r-k)n_r(t)n_{r-k}(t) - kn_k(t) \sum_{r=1}^{2r \leq k} (k-r)n_{k-r}(t) \right] \end{aligned} \quad (17.b)$$

$$k=M(t)-1,$$

$$\begin{aligned} \frac{dn_{M(t)-1}(t)}{dt} = & \frac{p_2}{M(t)} \sum_{r=1}^{M(t)-2} \alpha_{r,M(t)-r-1} r(M(t)-r-1)n_r(t)n_{M(t)-r-1}(t) \\ & - \frac{p_3}{M(t)} (M(t)-1)n_{M(t)-1}(t) \sum_{r=1}^{2r \leq M(t)-1} (M(t)-r-1)n_{M(t)-r-1}(t) \end{aligned}, \quad (17.c)$$

and

$$\frac{dN(t)}{dt} = p_1 - p_4 - \frac{p_2}{M(t)} \sum_k kn_k(t) \sum_{r=k+1}^{M(t)-k-1} rn_r(t) + \frac{p_3}{M(t)} \sum_k kn_k(t) \sum_{r=k+1}^{M(t)-k-1} (r+k)n_{r+k}(t), \quad (18.a)$$

$$\begin{aligned} \frac{dM(t)}{dt} = & p_1 - p_4 + \frac{p_2}{M(t)} \sum_{k=1}^{2k < M(t)} k[kn_k(t)]^2 \\ & - \frac{p_3}{M(t)} \sum_{k=2}^{M(t)-1} kn_k(t) \sum_{r=1}^{2r \leq k} (k-r)[(k-r)n_{k-r}(t) - \beta_{r,k-r} rn_r(t)] \end{aligned}, \quad \beta_{r,k-r} = \begin{cases} 0, & r = k - r \\ 1, & r \neq k - r \end{cases}. \quad (18.b)$$

The time-dependence of the concentration of clusters of size k , $Z_k(t) = \frac{n_k(t)}{N(t)}$, is connected to

the total number of agents in the system by equation $N(t) \sum_{k=1}^{M(t)-1} kZ_k(t) = M(t)$. The later sum

describes also the time dependence of the average group size, $M(t)/N(t)$.

We must resort to numerical methods for solving the problem. Note that one time step in this formulation corresponds to one update in the numerical simulation under time-dependent boundary conditions $n_1(t) \neq 0$ and $n_{M(t)}(t) \equiv 0$, and both non-zero initial cluster size distribution $n_k(0) \neq 0$ ($k=1, 2, 3, \dots, M(0)-1$), and when only free agents (monomers) are

present in the system at the initial moment $n_1(0)=M(0)$ and $n_k(0)=0$ ($k=2, 3, \dots, M(0)-1$), which are *a priori* defined, have been pointed out.

3. Results and discussion

The relative increase with time of the quantity of n -sized agents-clusters was numerically simulated in the case of no pre-existing clusters, i.e. when only free agents (monomers) are present in the system at the initial moment, and the results for $M=7$ ($n=2, 3, \dots, 6$) and $g_i/f_i = \{0.229, 1.458, 2.466, 2.835, 0.686\}$, $i=2, 3, \dots, 6$, and for $M=12$ ($n=2, 3, \dots, 11$) and $g_i/f_i = \{1.124, 1.381, 2.340, 0.322, 0.271, 2.043, 1.765, 0.764, 0.545, 1.298\}$, $i=2, 3, \dots, 11$, are shown in Figs 4 and 5, respectively. We can conclude that the functions that approximate analytically the curvatures presented in these figures are different. In the time limit $t \rightarrow 0$ these functions change according to $Z_n(t) \sim t^{n-1}$, regardless of the set of conditions used for $n=1, M$, and the size of the system. Meanwhile the evolution of any n -sized group of agents depends on the total number of agents in the system, M , and the particular shape of the time-dependent cluster size distribution, $Z_n(t)$, depends on f_i and g_i for the transitory period when a stable equilibrium is approached from the arbitrarily chosen set of initial conditions, but the relative time dependence of $Z_n(t)/X_n$ does not depend on f_i . For small clusters, e.g. $n=2, 3$ and even 4 for $M=12$, it is worth mentioning the existence of a linear or near linear evolution of the corresponding functions in short-time periods. These structures reach a stable equilibrium faster than any larger cluster ($n>4$). Therefore small groups of agents can quickly increase in the number and, finally, reach the value which corresponds to the equilibrium one.

Fig. 6 shows the results obtained for the time dependence of the average group sizes, $M/N(t)$, for $M=7$ and $M=12$. The sum $\sum_n nZ_n(t)$ was calculated between the limits 2 and $M-1$,

and it is represented for both sets of conditions for $n=1, M$ by full curves and broken curve,

respectively. The average group (cluster) size indicates that the system relaxes towards a constant value which depends on the type of conditions used and its size M , but the maximum value of $M/N(t)$ tends to nearly $(M-1)/2$, indicating the existence of a delicate balance between fragmentation and coagulation processes. In the time limit $t \rightarrow 0$ these functions change linearly with time. Meanwhile the average group size is constant for large t , and, taking into account the fact that the model is reminiscent of random growing systems and herding phenomenon, we can further use the relation obtained in this limit, when M/N is a constant too, between the number of clusters of size n and the exponent of the power law β [10], namely the attachment probability p of an agent to a cluster of size n , to carry out integration

of $\int_1^M nZ(n,t)dn$ upon consideration of n as a continuous variable and replacing the sum by the

corresponding integral. This leads to an equation $p \sim \int_1^M n^{1-\beta} dn$ or, after integration,

$p \sim \frac{M^{2-\beta} - 1}{2 - \beta}$. For any value $\beta \geq 3$, we get the condition $M^{2-\beta} \ll 1$ and find that $p \sim \frac{1}{\beta - 2}$. The

probability p varies, as it should be, in $[0, 1]$. The final equation for β takes the form

$\beta \sim 2 + \frac{1}{p}$, and this relation describes clearly the role of attachment probability in

“quantifying” economic growth. To match the empirical results [1, 13, 14] we require $p \approx 1/3$,

and obtain the expected distribution of returns with an exponent $\beta - 1 \sim 4$ [10].

Time evolution of the interacting n -sized agents-clusters was numerically simulated in the case of non-zero initial cluster size distribution for $M=7$, and the results for $n=2, 3, \dots, 6$ in both cases of arbitrary pre-existing clusters (full curves) and when only free agents are present in the system at the initial moment (broken curves) are shown in Fig. 7. We can conclude that the impact of pre-existing random generated clusters with different sizes $n=1, 2, \dots, 6$ on the non-stationary cluster size distribution in the short time limit depends on the size

of clusters, and it is stronger for smaller groups of agents. For large t the effect vanishes, and this is expected since the pre-existing clusters cannot affect the cluster distribution at equilibrium. They can only shorten the time needed for the establishment of the final distribution, and this fact can be clearly seen on the figure, regardless of the size n of clusters. Time dependence of the average group sizes, $M/N(t)$, represented in Fig. 8 for both types of initial conditions by full and broken curves respectively, also indicates that the system relaxes towards equilibrium faster in case of pre-existing random generated clusters. The sum $\sum_n nZ_n(t)$ was calculated between the limits 2 and $M-1$. In particular, it is important to note that these results give us more flexibility in choosing initial conditions, especially in case of evolution of large groups of agents in the time limit $t \rightarrow \infty$, i.e. for late stages of cluster formation, because the accuracy of analytical solutions will not be affected since the difference between particular results obtained by applying different initial conditions is insignificant.

Both non-zero initial cluster size distribution and when only free agents (monomers) are present in an open system at the initial moment have been studied. Preliminary numerical results show that, after a transient time, the system self-organizes into a critical metastable state that maximizes the entropy and exhibits a selective behavior of the non-stationary cluster size distribution in dependence of the total number of agents in the system and their transition probabilities. More specific data will be reported in the next paper. The obtained results indicate the importance of a concrete mechanism by which the agents-clusters change their size, regardless of the model used to study the kinetics of random evolution of market structure. In the time limits $t \rightarrow 0$ and $t \rightarrow \infty$, i.e. for early and late stages of cluster formation, they are expected to grow and decay mainly by gaining and losing single agents, because a relatively low cluster concentration at the early stage and an immobile, stable market at the late stage would require a single-agent mechanism of cluster formation. The advanced stage

could be characterized additionally by the merge of clusters of various sizes into a new bigger one (coagulation effect), and the contacts between them begin to have an increasingly important role in size changes. In this way a cluster can become larger not only by attaching a single agent, but also by merging with other clusters.

4. Conclusion

We have developed and solved numerically a kinetic model for the network of agents which evolves without preferential behavior in both cases of non-zero initial cluster size distribution and when only monomers are present in the system at the initial moment, as well as for an open system formed by time-dependent number of agents which behave with preferential attachment and fragmentation. Our results support the idea that some extremely fascinating universal aspects observed in the empirical analysis of finance fluctuations and economic organizations can really arise from an interplay between random growth and the complex structure of the system. Agents in a large collection interact with each other and form clusters of various sizes, and the size distributions have obvious implications for economic analysis. The application of the master equation is illustrated for some different examples, so the model could offer a general framework for the agent-based computational study of complex systems.

Acknowledgment

FP is grateful to the Royal Society for their financial support during his stay in Brunel University (UK), and would like to thank Prof. G.J. Rodgers for valuable discussions.

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Figure Captions

Fig.1. Schematic presentation of the possible changes in the size of a cluster of n agents. The quantity of n -sized clusters diminishes because of $n \rightarrow m$ transitions and increases due to $m \rightarrow n$ transitions, and these processes are represented by arrows leaving size n and those ending at size n , respectively.

Fig.2. Schematic presentation of the possible changes in the size of a cluster of n agents by nearest-size transitions (the Szilard model).

Fig.3. Schematic presentation of the possible changes in the size of a cluster of k agents. The quantity of k -sized clusters diminishes because of transitions **2** and **4**, and increases due to **1** and **3**, and these processes are represented by arrows leaving size k and those ending at size k , respectively. Broken curves represent schematically the boundary.

Fig.4. Relative time dependences of the quantity of n -sized agents-clusters in case of constant boundary conditions $Z_1=1$, $Z_M=0$, (a), and the time dependences of the concentration of clusters of size n for time-dependent $Z_1(t)$, (b), for $M=7$ ($n=2, 3, 4, 5$ and 6 , from top to bottom). The insertions represent these results for short-time periods.

Fig.5. Relative time dependences of the quantity of n -sized agents-clusters for $M=12$ ($n=2, 3, \dots, 11$, from top to bottom); the case of constant boundary conditions.

Fig.6. Time dependence of the average group sizes for $M=7$ and $M=12$. Full curves represent the results in case of constant boundary conditions, and broken curve that for time-dependent $Z_1(t)$. The sum was started from $n=2$ in both cases.

Fig.7. Time dependences of the concentration of clusters of size n in case of arbitrary pre-existing clusters (full curves) and when only free agents are present in the system at the initial moment (broken curves) for $M=7$ ($n=2, 3, 4, 5$ and 6 , from top to bottom).

Fig.8. Time dependences of the average group sizes for $M=7$. Full curve represents the result in case of pre-existing random generated clusters with different sizes $n=1, 2, \dots, 6$, and broken curve that for only $Z_1(0) \neq 0$.

Fig. 1. Paladi

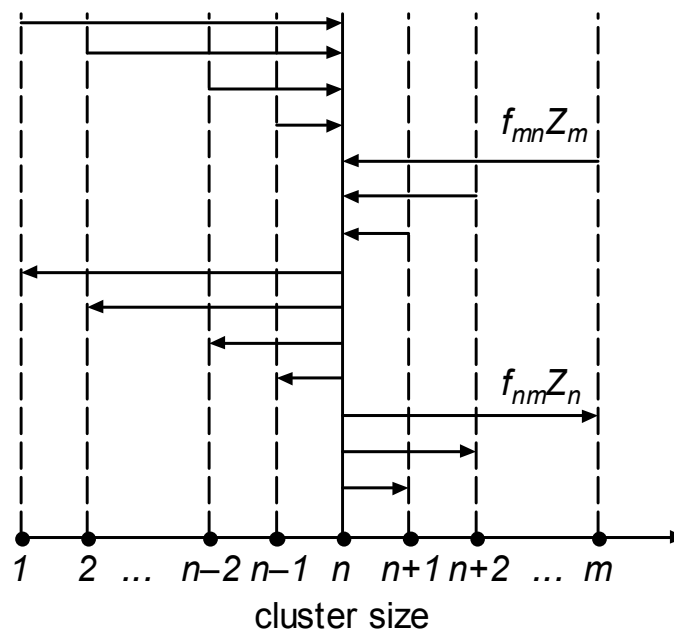


Fig. 2. Paladi

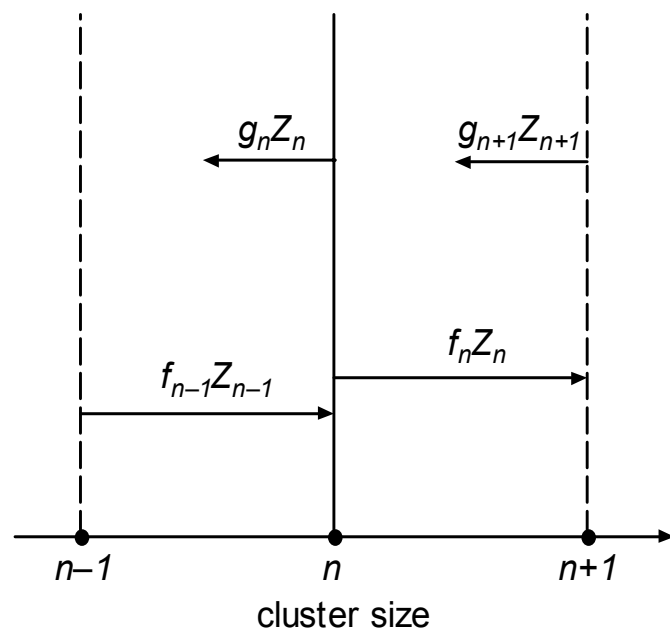


Fig. 3. Paladi

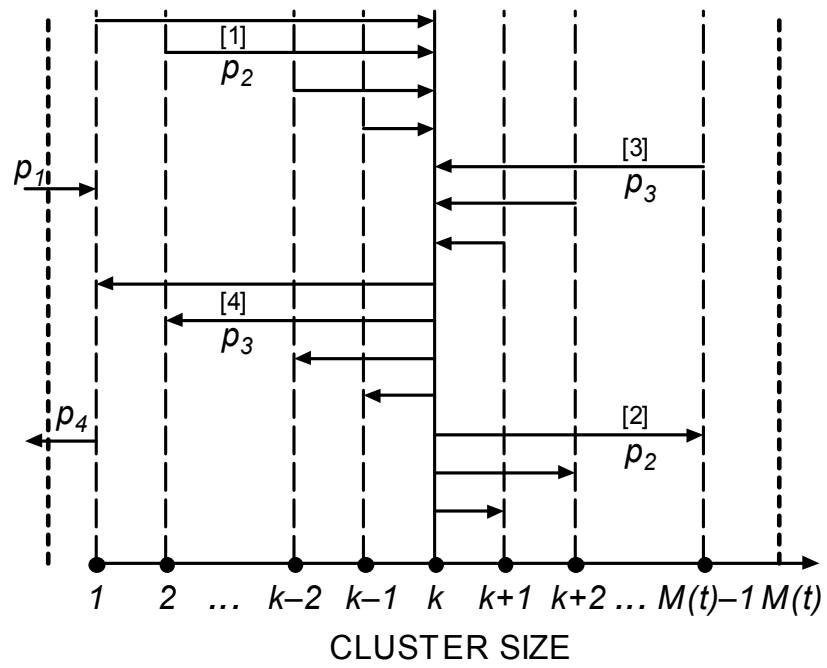


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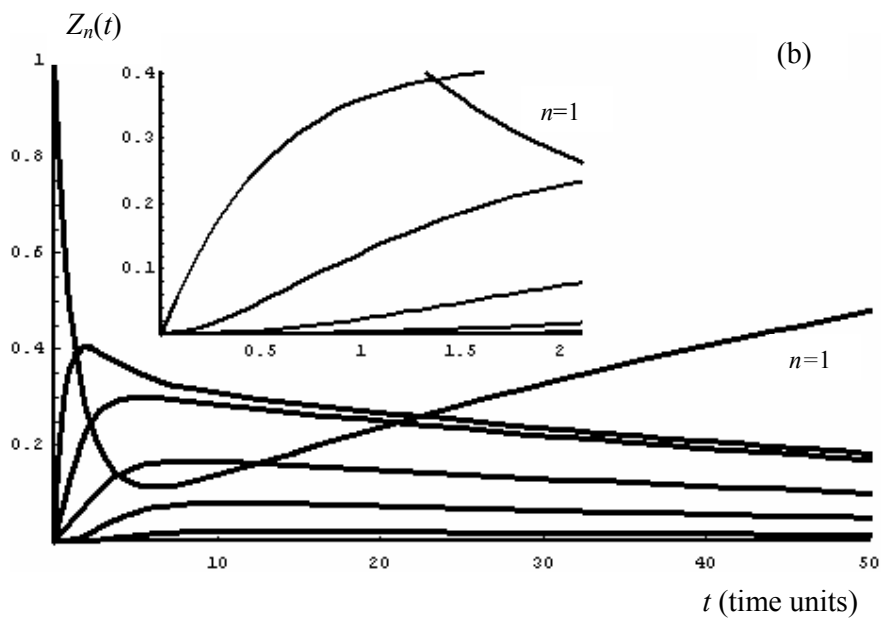
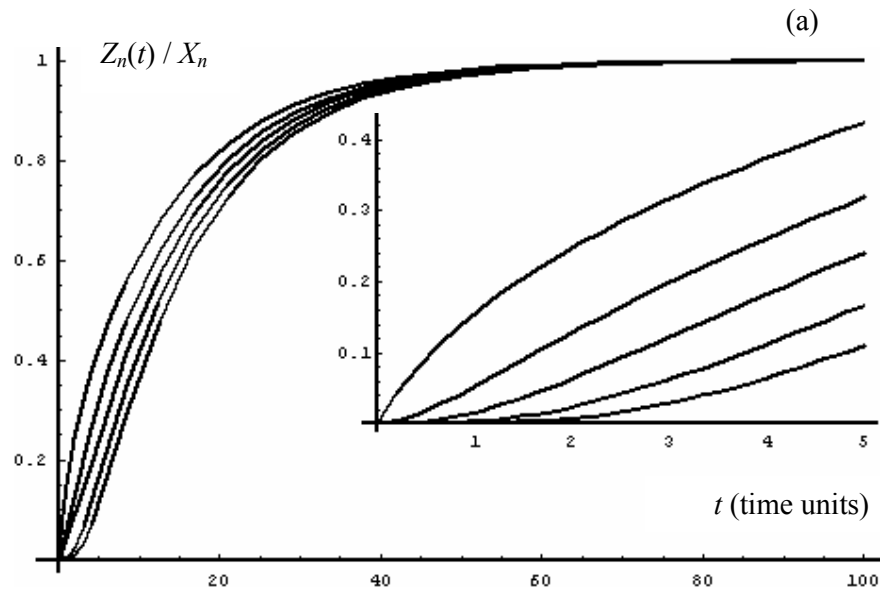


Fig. 5. Paladi

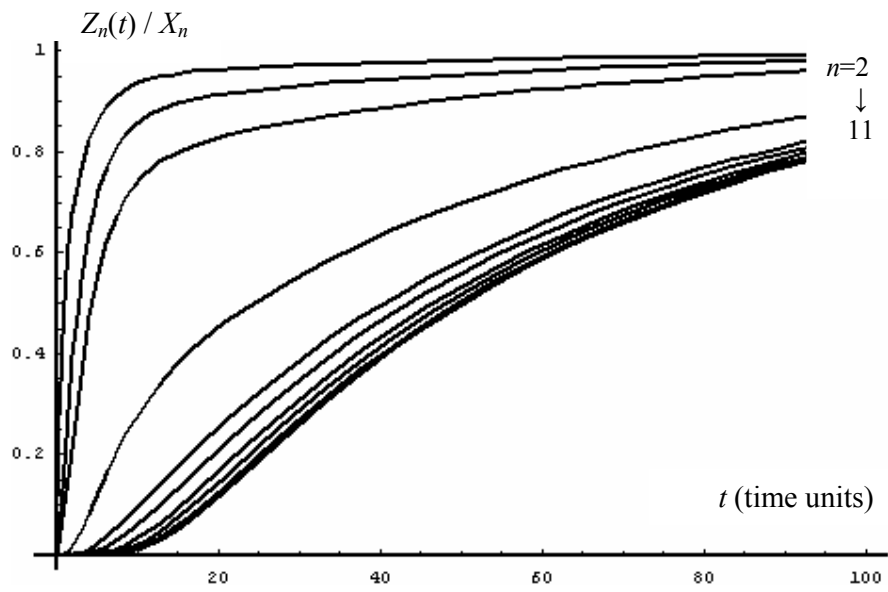


Fig. 6. Paladi

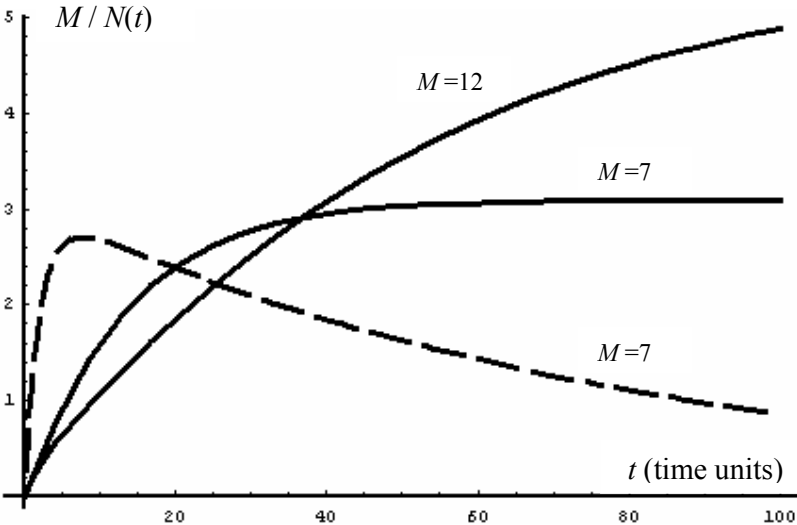


Fig. 7. Paladi

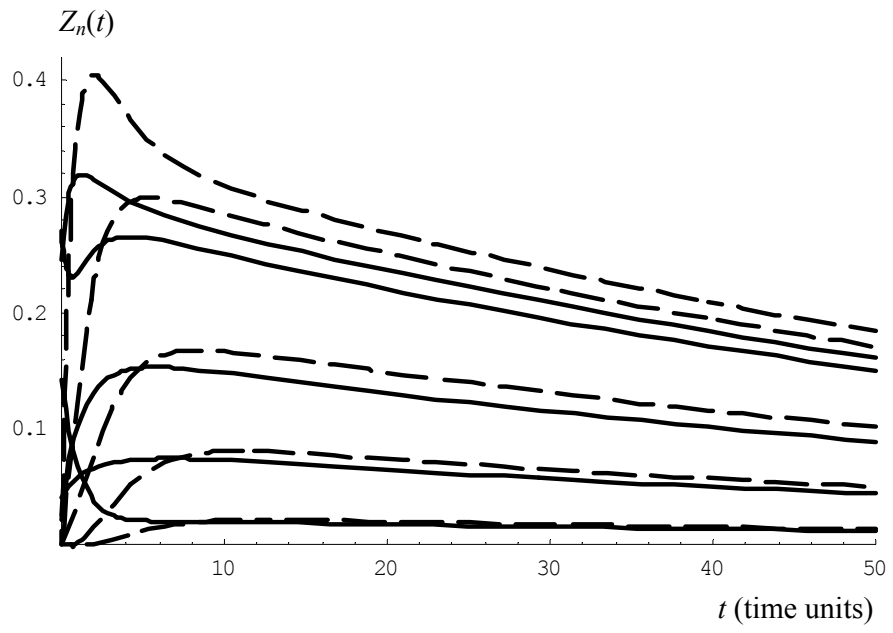


Fig. 8. Paladi

