Explaining the Distribution of Firms Growth Rates*

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Abstract

Recent empirical analyses on aggregated datasets have revealed a common exponential behavior in the shape of the probability density of the corporate growth rates. In this paper we present clearcut evidence on this topic using more disaggregated data.

Then we propose a very simple model, the first to our knowledge, that, under rather general assumptions, provides a robust explanation for the observed regularities.The mechanism is based on the idea that the firm ability of catching new business opportunities increases with the number of opportunities already exploited. A theoretical result is presented for the limiting case in which the number of firms and opportunities go to infinity. Moreover, using simulations, we show that even in a moderately small industry the agreement with asymptotic results is almost complete.

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1 Introduction

One of the most traditional problem in the Industrial Organization literature concerns the statistical properties of the size of firms and its dynamics.

Early investigations were primarily focused on two aspects of the general problem, namely the analysis of the size distribution and the characterization of firms growth dynamics in terms of autoregressive stochastic processes. The log-normal character of the upper tail of the size distribution was quite unanimously considered the natural benchmark while the dynamic analysis has mainly focused on the application of unit-root tests to the growth rates, to verify the Gibrat hypothesis (Gibrat, 1931) of random-walk growth and to find possible violations (in the enormous body of contributions see for instance Dunne et al. (1988); Evans (1987); Hall (1987)).

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These early investigations were conducted over datasets at a high level of aggregation, typically including large firms operating in very different sectors. For instance Hart and Prais (1956) study the distribution of the whole U.K. manufacturing industry while Simon and Bonini (1958) explore the size distribution of the top manufacturing firms of the U.S. economy, across all the sectors. A common source of problems in considering such aggregate data is the possibility of introducing statistical regularities that are only the result of the aggregation process (e.g. via Central Limit Theorem), thus concealing the true properties of the dynamics of business firms that are active in specific sectors. Indeed Hymer and Pashigian (1962), analyzing more disaggregated data, find a high heterogeneity in firms size distributions across different sectors. They conclude that it is quite unclear whether any "stylized fact" regarding the size distribution actually exists. As far as the validity of the Gibrat's hypothesis is concerned, the conclusions of these works are variegated, if not contradictory (for a critic review see Sutton (1997)).

Moving from the foregoing traditional econometric issues, a new strand of analysis recently emerged that proposes a wider statistical characterization of companies growth dynamics. Following these lines of research the present paper, extending preliminary results reported in Bottazzi and Secchi (2003), analyses the growth rates distribution of business firms in the Italian manufacturing industry using data disaggregated by sectors. The results are impressive: the growth rate probability density shows a basically identical tent-shape in all the sectors under study. The same tent-shape was found to characterize growth rate density in U.S. manufacturing industry by Stanley et al. (1996) and in the world-wide pharmaceutical industry by Bottazzi et al. (2001).

The robustness of this empirical finding constitutes an interesting theoretical issue that remains, however, completely unexplained by the few standard models present in the literature. In our opinion the reason for that can be traced back to the presence, in those models, of noticeable weaknesses. First of all, from the seminal work of Gibrat (Gibrat, 1931) to the more recent contributions of Geroski (2000) and Amaral et al. (2001), many models do not assume any interdependence between the histories of different firms. The dynamics of each firm is a stochastic process, encompassing growth, diversification, entry and exit, that, nevertheless, does not keep in consideration the behavior of the other firms. Each firm acts as if it was a monopolist in a sector whose dynamics can be represented simply with an exogenous expansion (or contraction) of the demand. A different kind of models, originally proposed by Simon (see Ijiri and Simon (1977)) and later reconsidered by Sutton (1998) make the assumption that there is a finite set of pre-existing growth opportunities (or equivalently, a constant arrival of new opportunities) and that firms growth process is conditioned by the number of opportunities they are able to catch. Roughly speaking, one could say that these models, generically known as "islands models", try to provide a first account of the competitive behavior among firms based on the idea that firms need to seize the most out of a scarce resources. Nevertheless also these models fail to explain the empirical shape of the growth rate density.

A more structured approach to the inclusion of a competitive dimension in the description of industrial dynamics has been a main feature of the vast body of models developed in the last 20 years. Consider, for instance, the notion of "noisy selection" in Jovanovic (1982) or the one of "Schumpeterian competition" in Nelson and Winter (1978). Even if these models were successful in bringing a more plausible microeconomic foundation in the description of business firms dynamics, they came to little help in explaining the statistical properties of growth rates since they seem to completely ignore these empirical issues.

In the present paper we build a simple mechanism of firm dynamics where a stylized idea of competition is introduced: indeed, also in our model, rephrasing Nelson and Winter (1978), luck is the principal factor that finally distinguishes winners from losers among the contenders. This mechanism is intended as a contribution to the Simon inspired tradition, at least in its ambition to meet both the requirements of simplicity and generality. We introduce a stochastic description where each firm is considered a different realization of the same process. Similarly to what happens in the island models, the symmetry is however broken at the aggregate level: the total growth of the whole population of firms is bounded by a finite set of sector-specific opportunities.

The novelty of our approach resides in the way in which we describe the random distribution of opportunities among firms. In the existing formulations the splitting procedure carries no history and at each competitive round each firm possesses the same probability of seizing market opportunities. Conversely, our assignment procedure allows to represent self-reinforcing mechanisms whereby the probability for a given firm to catch a new opportunity positively depends on the number of opportunities already caught.

This different specification of the opportunities assignment allows a much better agreement with empirical findings. No other model in the literature is indeed able to reproduce the characteristic shape of the growth rates distributions observed in empirical analyses. Moreover, the extreme simplicity of the proposed mechanism suggests that it can plausibly be used as a representation of the core behavior of quite diverse models, aimed at the description of different industrial sectors and accounting for different aspects of the economic dynamics, like, for instance, technological spillovers, increasing returns in research activity or positive network externalities.

The remainder of this paper is organized as follows. Section 2 examines the empirical data. Section 3 proposes a new and rather general stochastic model of the growth process while Section 4 continues its analysis and compares its features with the empirical findings. The generality and robustness of the model is provided by the theorem proved in Section 5. Section 6 draws some conclusions and briefly comments on the need for further theoretical research.

2 Empirical evidence

Some years ago in a series of papers based on the COMPUSTAT database Stanley et al. (1996) and Amaral et al. (1997) analyzed the probability distribution of the (log) growth rates of publicly traded U.S. manufacturing firms. These studies were performed using observations in the time frame $1974-93$ and on companies with primary activity belonging to the SIC code range $2000-3299$. Different lines of business inside the same multi-activity firm were completely aggregated. According to these analyses, the growth rates of business firms, when aggregated across all the sectors, appear to robustly follow a characteristic tent-shape probability density. Hence a Laplace (symmetric exponential) functional form

$$f_{\mathrm{L}}(x; \mu, a) = \frac{1}{2a} e^{-\frac{|x-\mu|}{a}} \tag{1}$$

was proposed in order to describe the empirical observations. More recently Bottazzi et al. (2001), studying the PHID database[1], found the same characteristic shape for the empirical

[1] The Pharmaceutical Industrial Database (PHID) has been developed by F. Pammolli, University of Florence.

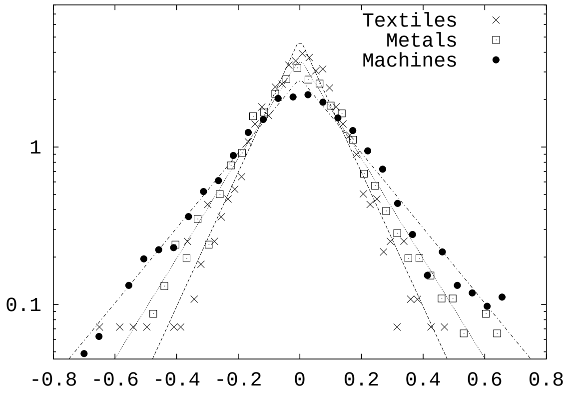

Figure 1: Binned empirical densities of the growth rates for the three sectors of textiles finishing, treatment and coating of metals and special purpose (metallurgy, mining, chemistry ...) machinery. The distribution pooled over all the years is reported.

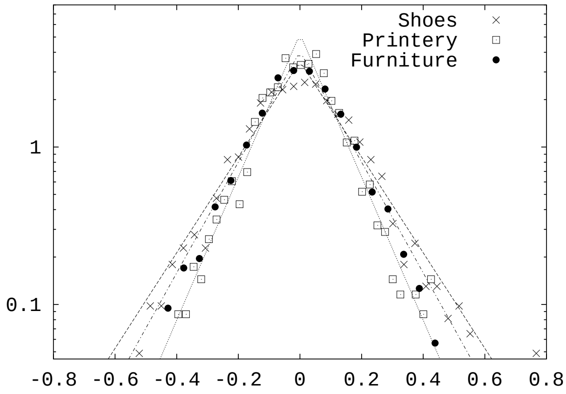

Figure 2: Binned empirical densities of the growth rates for the three sectors of footwear, printing and furniture. The distribution pooled over all the years is reported.

density of the growth rates of the largest worldwide companies in the pharmaceutical industry.

The similarity across these early studies naturally leads to the question of how general this tent-shape character is when different industries or countries are taken in consideration. What is more, these studies where focused on very large multi-plants and/or multinational companies and, in particular for the COMPUSTAT based analysis, data were aggregated across many distinct sectors. Hence a further possible issue concerns the robustness of this finding when smaller firms and disaggregated data are analyzed. In the present section we address these two issues. We study a unique dataset, that includes a large part of the Italian manufacturing industry, to add new evidence to the original finding. The analysis is conducted sector by sector in order to check to what extent the mentioned finding survives at a more disaggregated level.

The analysis presented here draws upon the MICRO.1 databank developed by the Italian Statistical Office (ISTAT)[2]. MICRO.1 contains longitudinal data on a panel of several thousands of Italian manufacturing firms, with 20 or more employees, over a decade. For statistical reliability we restrict our analysis to the period $1989-96$ and to the sectors with more than 44 firms. Under these constraints, the number of 3 digit sectors under study is reduced from 97 to 55.

In what follows we use, as a definition of firms' size, the total sales. Let $S_{i,j}(t)$ represents the sales of the i-th firm, belonging to the j-th sector, at time t. Here $j \in \{1,\ldots,55\}$ and if N_j is the number of firms in the j-th sector, one has $i \in \{1,\ldots,N_j\}$. In order to eliminate possible trends, both sector specific and industry-wide, we consider the normalized (log) sales

$$s_{i,j}(t) = \log(S_{i,j}(t)) - \frac{1}{N_j}\sum_{i=1}^{N_j}\log(S_{i,j}(t)) \tag{2}$$

subtracting from the (log) size of each firm the average (log) size of all the firms operating in

[2]The database has been made available to our team under the mandatory condition of censorship of any individual information.

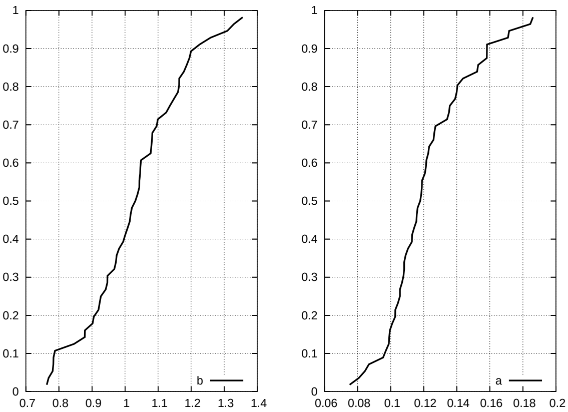

Figure 3: Distribution of the Subbotin shape parameters b and a estimated using maximum likelihood over the sectors population. The values for the different sectors are reported in Table 1.

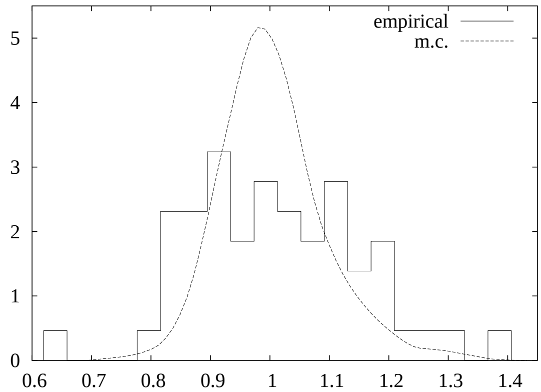

Figure 4: The binned empirical density of the b parameter values estimated using maximum likelihood over the different sectors. A Monte Carlo computation of the theoretical distribution under the hypothesis of Laplace-distributed growth rates is also shown.

the same sector. The (log) growth rate is then defined according to:

$$g_{i,j}(t) = s_{i,j}(t+1) - s_{i,j}(t) \quad . \tag{3}$$

Notice that from (2) the distribution of the g's is by construction centered around 0 for any t.

As a first qualitative investigation one can simply plot the observed densities for different sectors. Fig. 1 and Fig. 2 show the growth rates densities for six different three digit sectors chosen because both numerous and structurally diverse. The activity indeed ranges from footwear production to the treatment of metals for industrial purposes. All the 7 years of data are pooled together under the assumption of stationarity of the growth process. For each sector the Laplace density estimated via maximum likelihood is also shown. As can be seen, these fitted densities well describe the observations.

In order to quantify the agreement with the Laplace and to give a synthetic account of its robustness and generality in describing empirical densities we follow a parametric approach. We consider a flexible family of probability densities,known as the Subbotin family (Subbotin, 1923), already used in Bottazzi et al. (2002), that includes as a particular case the Laplace. This family is defined by 3 parameters: a positioning parameter μ, a scale parameter a and a shape parameter b. Its functional form reads:

$$f_S(x) = \frac{1}{2ab^{1/b}\Gamma(1/b+1)}\, e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^b} \tag{4}$$

where $\Gamma(x)$ is the Gamma function. The lower is the shape parameter b, the fatter are the density tails. For $b < 2$ the density is leptokurtic and is platikurtic for $b > 2$. It is immediate to check that for $b = 2$ this density reduces to a Gaussian and for $b = 1$ to a Laplace. For each sector we compute the density that better fits the data among those belonging to this family. We estimate the a and b parameters for each sector maximizing the likelihood of observations, since the parameter μ is set to 0 by the normalization in (2).

The distribution function of the b parameter estimates over the 55 sectors is reported in Fig. 3. The values for specific sectors can be read from Table 1. Notice that almost 80% of the

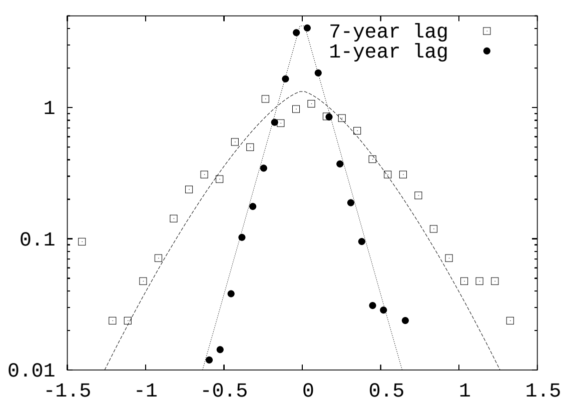

Figure 5: Binned empirical densities of the growth rates, evaluated with two different time horizons (1 and 7 years), for the Furniture sector (ATECO 361) in the Italian manufacturing industry.

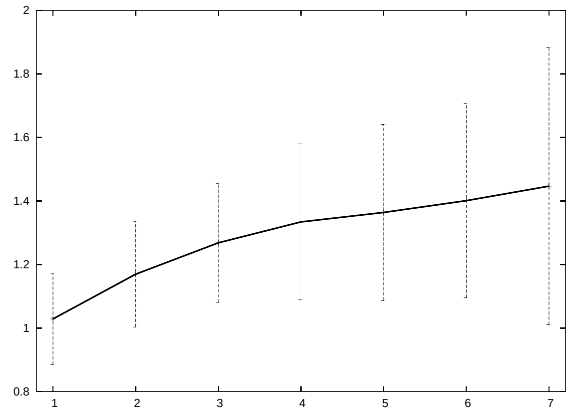

Figure 6: Estimated Subbotin shape parameter b of the growth rates distribution for different time horizons. The value reported is the average over all the sectors.

b values range between .85 and 1.2, suggesting that (1) provides a very good approximation of the empirical densities. However, noticeable departure are present. Regarding this point it is important to understand whether these observed departures of the b parameter from the Laplace value of 1 can be explained by statistical errors or, on the contrary, bear some economic content.

To answer this question it is useful to revert to Monte Carlo techniques. We assume as null hypothesis that the 55 sectors are distinct and independent realizations of the same Laplace model but with different values for the number of firms and for the parameter a in (1). We use the simple relation between the standard deviation σ of the growth rates distribution and the parameter a, namely $\sigma = \sqrt{2}\, a$, to obtain an estimation for the value of the latter in each sector. Once these parameters are known, it is possible to compute, using repeated simulations, the probability to obtain a given value of b via maximum likelihood procedure. In Fig. 4 the binned empirical density of the b values is shown together with the theoretical density obtained with Monte Carlo simulations under the null hypothesis of Laplace distributed growth rates.

As can be seen, the agreement is substantial, even if far from perfect. The conclusion that can be drawn is mixed: quite plausibly there are "essential" deviations, not statistical in nature, from the Laplace model, but these deviations affect a small part, probably less then 10%, of the sector under studies.

Incidentally, notice that also the distribution of the "scale" parameters a, estimated via likelihood maximization and reported in Fig. 3, possesses a remarkably narrow support. This evidence, again, suggests a quite strong similarity among the growth rates densities in different sectors. More important, due to this property, we observe the emergence of the Laplace distribution in our database even when we consider data aggregated over all sectors, in accordance with the evidence shown in Stanley et al. (1996).

We conclude our analysis of the firms growth rates by looking at their structure on a longer, multi-year, time horizon. In line with previous notation let us define the growth rate on a T year period as:

$$g_{i,j}(t;T) = s_{i,j}(t+T) - s_{i,j}(t) \quad . \tag{5}$$

When $T = 1$, (5) reduces to the one year growth rates defined in (3). Using again maximum likelihood estimation one can compute the value of the a and b parameters in each sector at different T. As can be seen from Fig. 6 the average value of the b parameters across all the sectors, that is near to 1 when $T = 1$, steadily increases for longer intervals. This implies that the typical shape of the growth rates density becomes more similar to a Gaussian when longer time horizons are considered. An example of this progressive reduction of density tails is shown in Fig. 5. This effect can be considered natural if the firm growth shocks relative to different years were independent and, consequently, the progressive normalization of the growth rate density wasan effect of the Central Limit Theorem (CLT). Notice, however, that the slope of the curve in Fig. 6 seems to decrease rapidly as T increases suggesting that the asymptotic value of b can be quite below the expected value of 2. The time horizon of our database is however too short to allow a reliable discussion of this point. As one consider longer time lag, the number of available observations decreases and the statistics become so noisy that it is impossible to conclude if some further effect, apart the CLT, is at work here. Moreover, the heterogeneity of the autocorrelation coefficients of the growth rates in different sectors (see Table 1) tends to complicate the matter.

In conclusion, the empirical findings of this section can be summarized as follows. First, the tent-shape distribution of firms growth rates appears as a general feature of industrial data; its validity is, indeed, confirmed also for the Italian manufacturing industry. Moreover, the Laplace density constitutes a good approximation of the observed densities not only when large firms and/or aggregated data are considered, but also for medium sized firms and at a disaggregated level. Given the importance of disaggregated analysis in order to eliminate the possible effects of the Law of Large Numbers, our findings give more solid foundations to the hypothesis of the existence of a general shape for the growth rates distribution of firms. Second, this characteristic shape tends to disappear when longer time horizon are considered, as suggested by the Central Limit Theorem under the hypothesis of independent growth events.

In the next Section we propose a general mechanism that provides a simple and, in our view, economic sounding explanation for these empirical findings.

3 A model of firm growth

In the broad literature about the growth dynamics of business firms, there exists a well established tradition that describes the modification of firm size, over a given period of time, as the cumulative effect of a number of different "size" shocks generated by the diverse accidents that affected the firm history in that period (see, among many contributors, Kalecki (1945); Ijiri and Simon (1977); Steindl (1965) and more recently Amaral et al. (2001); Geroski (2000); Sutton (1998)). Since these models of growth are usually described in terms of multiplicative processes it is natural to use logarithmic quantities to describe the "status" of a given firm. Consider a firm i and let $s_i(t)$ be its (log) size at time t. One can write

$$g_i(t;T) = s_i(t+T) - s_i(t) = \sum_{j=1}^{G_i(t;T)} \epsilon_j(t) \tag{6}$$

where the firm growth in the period $[t, t+T]$ is described as a sum of $G_i(t;T)$ "shocks" each one having a finite effect ϵ on firm size. In empirical studies, the time lag T can range from 3 months for quarterly data, to $30 - 50$ years for the longest databases. In the oldest and probably most controversial model of Gibrat (Gibrat, 1931) the shocks are assumed independent random variables, so that the firms growth is described as a geometric Brownian motion. The growth rates associated to different non-overlapping time periods are independent and when the number of shocks $G_i(t;T)$ becomes large the rate of growth $g_i(t;T)$ tends, for the Central Limit Theorem, toward a normal distribution.

However we showed in the previous Section that this is not the case in the real world: in three very different databases, at least when yearly data are considered, a Laplace distribution fits much better than a Gaussian the data.

Since the Gibrat's model cannot yield an equilibrium distribution of the growth rates that resembles the observed one, we are led to conclude that some of the assumption adopted is not appropriate.

Probably the most noticeable drawback of the Gibrat's idea resides in the implicit assumption that companies growth process are completely independent. This is equivalent to assume that any form of competition is absent even among firms operating in the same sector and selling on the same market. This assumption sounds rather implausible. To this respect, a different theoretical tradition, dating back to the early work of Simon and recently renewed by Sutton, has been developed with the aim of introducing in the family of Gibrat-type stochastic models of growth a stylized description of competition and entry dynamics.

These models, known as "islands models", assume the existence of a finite number of business opportunities available to firms. All the firms, operating in a number of independent sub-markets (islands), take up these opportunities and their growth process is measured by the number of them they are able to take. They depart from the Gibrat tradition with respect to two points. First, even if each business opportunity concerns only one firm, the symmetry of the growth process of different firms is broken, in the aggregate, by the fact that the business opportunities are limited. Second, there is always a finite probability that business opportunities are catch by new firms. In these models the constant G_i is reinterpreted as a stochastic variable, representing the outcome of a random assignment procedure of business opportunities among incumbent and entrant firms.

It turns out that even the "island models" fail to account for the observed tent-shaped density of growth rates. Indeed, if one switches off the entry dynamics, that is explicitly ignored in the empirical investigations presented in the previous Section, these models again reproduce a Gaussian growth density. The reason comes from the assumed equiprobability of incumbent firms to capture new business opportunities when the process is described in terms of logarithm. In this case the unconditional distribution of G_i for a given firm is binomial; hence in the limit of many small opportunities one obtains again, via Law of Large Numbers, a Gaussian form.

The remainder of this Section is devoted to presenting a model showing that if one modifies this basic assumption of "equal assignment probabilities" of each opportunity the shape of the growth rates distribution changes and is no longer a Gaussian. We basically retain the "island model" approach and describe the growth of a firm as a two steps process. In the first step there is a random assignment among firms of a given number of business opportunities leading to a possible realization of the random variables G_i. These opportunities represent all the variegated "accidents" that can plausibly affect the history of a business firm, encompassing, for instance, the exploitation of technological novelties, the reaction to demand shocks and the effects of managerial reorganizations. The assigned business opportunities, in the second step, act as the source of micro-shocks affecting the size of the firm according to equation (6).

The first step of our model is based on a simple stochastic partition of a finite number of business opportunities, say M, among a population of N identical firms. Instead of assuming, as common in the cited models, that the assignment of each opportunity is an independent

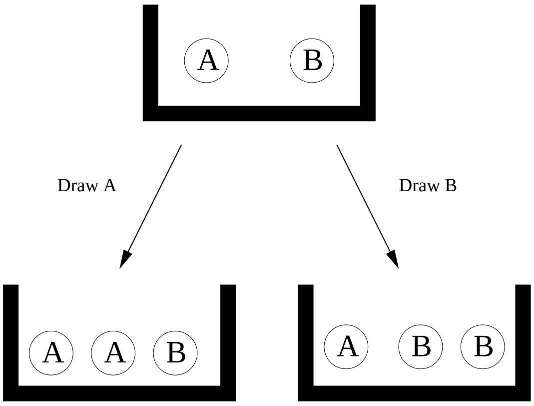

Figure 7: First drawn from a urn with two colors labeled A and B. After the extraction the state of the urn depends on which color was drawn.

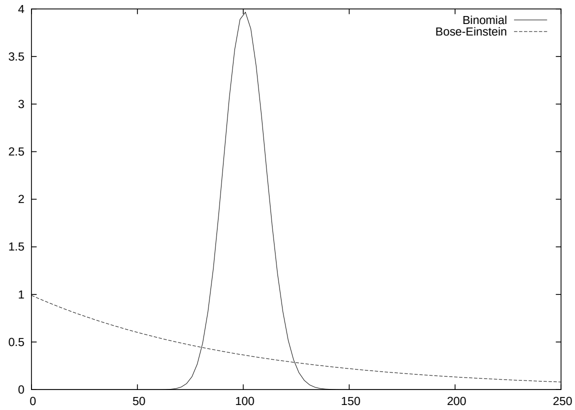

Figure 8: The comparison of the Bose-Einstein (9) with a corresponding Binomial distribution with $N = 100$ and $M = 10,000$.

event with constant probability $1/N$, we introduce the idea of "competition among objects whose *market success*... [is] cumulative or self-reinforcing" (Arthur , 1994, 1996). Economies of scale, economies of scope, network externalities and knowledge accumulation are just few examples of possible underlying economic mechanisms that are able to generate these effects of positive feedbacks within markets, businesses and industries. We model this idea with a process where the probability for a given firm to obtain new opportunities depends on the number of them already caught. Such a procedure of sequential assignment of M business opportunities among N firms is easily described using a Polya's urn scheme.

Consider an urn containing balls of N different types. In this urn there is one ball for each type and each type represents a specific firm. A ball is drawn at random, then it is replaced and, moreover, 1 ball of the type drawn is added. Another random drawing is made from the "new" urn containing one more ball and this procedure is repeated M times. It is straightforward to notice that in this way we introduce the desired effect that the drawing of either type increases the probability of the same type at the next drawing (See Fig. 7 for a simple graphical exemplification of a first step of this procedure). We can now interpret the drawing of the ball of type i as the assignment of one opportunity to firm i. In this context the outcome of each process of assignment of all the opportunities among firms is completely described by the occupancy N-tuple $(m_1, m_2, \ldots, m_N)$ where $\sum_{j=1}^{N} m_j = M$. The probability of obtaining a particular N-tuple (Feller (1968), p.120) is given by:

$$P\Big((m_1, m_2, \ldots, m_N)\Big) = \frac{1}{\binom{N+M-1}{N-1}} \quad . \tag{7}$$

The conditional probability of the same N-tuple given that h opportunities have already been assigned to a given firm, can be derived from (7) and reads:

$$P\Big((m_2, \ldots, m_N)|m_1 = h\Big) = \frac{1}{\binom{N+M-h-2}{N-2}} \quad . \tag{8}$$

It is now easy to compute the marginal probability that a given firm obtains exactly h

opportunities:

$$P(h;N,M)=\frac{P\Big((m_1,m_2,\ldots,m_N)\Big)}{P\Big((m_2,\ldots,m_N)|m_1=h\Big)}=\frac{\binom{N+M-h-2}{N-2}}{\binom{N+M-1}{N-1}} \tag{9}$$

which is the well known Bose-Einstein statistics[3]. To give an idea of the outcome of the previous assignment procedure in Fig. 8 we compare the Bose-Einstein distribution with the binomial obtained when each opportunity is assigned with the same probability. The "clustering" effect is evident in the fat tailed nature of the Bose-Einstein distribution, that suggests an increased probability for a given firm of obtaining a large number of opportunities. Furthermore this distribution possesses a 0 modal value in sharp contrast with the M/N value generated by an independent and equiprobable opportunities assignment.

The procedure just described provides a particular partition of M opportunities among N firms summarized by the N-tuple $(m_1,\ldots,m_N)$. As already mentioned, these business opportunities can be thought of as the source of micro-shocks affecting the size of firm. We make no assumptions on the actual nature of these shocks and we want to relate "opportunities" to "growth" in the simplest way. Hence, we assume that these micro-shocks are randomly and independently drawn from a distribution with fixed variance and, since we are interested only in the distribution of the relative growth rates, zero mean. The total growth of firm i is obtained adding $m_i(t)+1$ independent micro-shocks, $m_i(t)$ assigned by the Polya process and the 1 already in the urn at the beginning of the extractions. If $s_i(t)$ stands for the (log) size of firm i at time t, the growth equation reads

$$s_i(t+1)=s_i(t)+g_i(t) \qquad g_i(t)=\sum_{j=1}^{m_i(t)+1}\epsilon_j(t) \tag{10}$$

where ϵ are i.i.d. with a common distribution $F(\epsilon;v_0)$ with variance v_0 and mean 0. Notice that the random growth rates g_i are identically, but not independently, distributed across firms, due to the global constraint $\sum_i m_i=M$. The unconditional probability distribution implied by the assignment procedure in (9) is

$$F_{\mathrm{model}}(x;N,M,v_0)=\sum_{h=0}^{M}P(h;N,M)\;F(x;v_0)^{\star(h+1)} \tag{11}$$

where $F(x;v_0)^{\star h}$ stands for the h-time convolution of the micro-shocks distribution (i.e. the distribution of the sum of h micro shocks). The average number of opportunities per firm is M/N and the distribution of growth rates g has mean 0 and variance $v=v_0M/N$.

At this point it is useful to clarify a few points about the assumptions just considered. First, concerning the zero mean hypothesis, notice that the choice of a distribution with a non-zero mean m_0 would simply introduce an industry-wide growth trend proportional to Mm_0 which is irrelevant if one is interested in the distribution of growth rates[4]. Second, both the hypotheses of no correlation among micro-shocks and of constant variance in their distribution are working hypotheses introduced to keep the discussion clearer and can be relaxed, for instance introducing a mild correlation among micro-shocks or introducing a

[3]This statistics is mainly used in physics where it describes the peculiar thermodynamic behavior of a large family of subnuclear particles.

[4]Notice that in all the empirical studies cited in Section 2 the actual variable under study is the "normalized" growth rate, i.e. the growth rate of market shares.

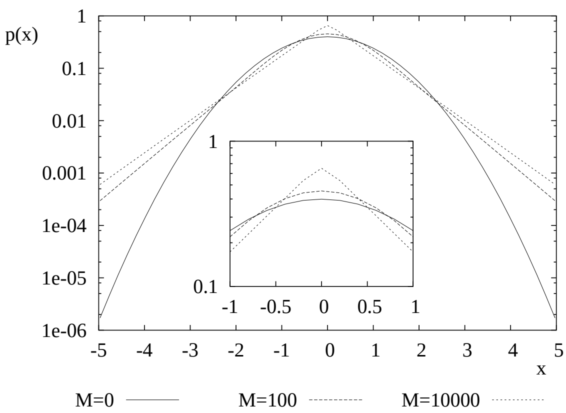

Figure 9: Growth rates probability density for $N = 100$ and different values of M. The inset shows a particular of the central region.

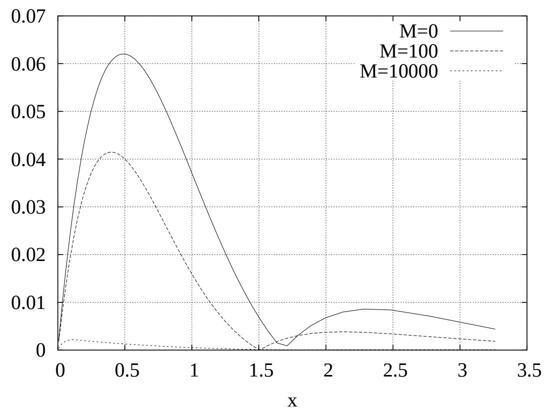

Figure 10: Absolute deviation $|F_{\mathrm{model}}(x) - F_{\mathrm{L}}(x)|$ as a function of x for $N = 100$ and different values of M.

random variance extracted from a given distribution. Finally, the choice of the shape for the micro-shocks distribution is, as we show later, completely irrelevant in all the cases of interest.

4 The source of the tent-shape

The mechanism presented in the previous Section is rather parsimonious in terms of the required parameters. Indeed it is able to provide a uniquely defined distribution for the firm growth rates when only three components are specified: the number of firms operating in the market N, the total number of "business opportunities" M representing the "sources" of the firms growth events and the effect that these events have on the size of the firm, captured by the micro-shocks probability distribution $F(x; v_0)$.

In this Section we analyze extensively the properties of the mechanism presented. Our aim is to understand under which conditions this mechanism is able to reproduce the empirical regularities described in Section 2. We show that when the number of firms N and the average number of micro-shocks per firm M/N become large, the growth rate distribution obtained from (11) progressively approaches a Laplace distribution.

In order to simplify the discussion, we assume that the micro-shocks are normally distributed, with unit variance $v_0 = 1$, i.e. $F(x; 1) = N(x; 0, 1)$. The micro-shocks density then becomes $f(x) = F'(x) = \exp(-x^2/2)/\sqrt{2\pi}$. This assumption is made only to keep the discussion easier, and we show in the next Section that our conclusions are largely independent from the actual shape of the micro-shock distribution.

We can start our analysis with an example. Consider a sector with a reasonable number of firms, let say 100. This number is more or less of the same order of the population size in the manufacturing sectors analyzed in Section 2. Now suppose that no assignment of opportunities is performed, i.e. that $M = 0$. In other terms, each firm ends up with just one shock, the one originally put in the urn. Since the micro-shock distribution is $N(x; 0, 1)$, and exactly one shock is assigned to each firm, the observed growth rates distribution will have the same normal form. A picture of the associated density is reported in Fig. 9 with the label $M = 0$. The log scale on the y axis makes its parabolic shape clear. Now suppose instead to have a positive number of opportunities, for instance suppose that $M = 100$, so that the average number of opportunities per firm is now increased to 2. Again, suppose that the micro

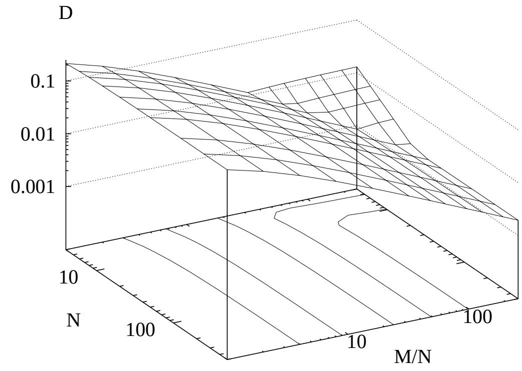

Figure 11: Absolute deviation D as a function of the number of firms N and the average number of micro-shocks per firm M/N. Micro-shocks are normally distributed.

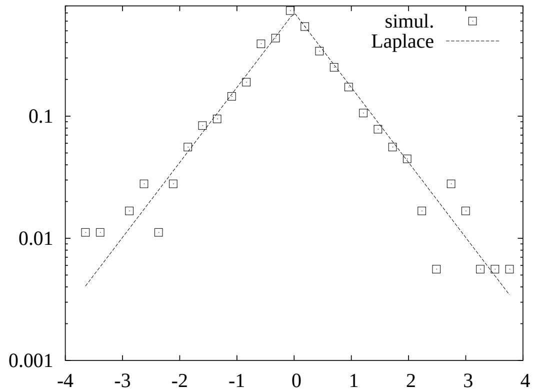

Figure 12: Growth rates probability density with $\lambda = M/N = 16$, $N = 100$ and normally distributed micro-shocks. A pool of 7 independent realizations in considered. The theoretical Laplace density with unit variance ($a = 1/\sqrt{2}$) is also shown.

shocks are distributed according to $N(x; 0, 1/2)$. This means that if the opportunities would be assigned independently, the firm growth rates would be the sum of two (the average number of shocks) normal variates with variance 1/2, that is a normal variate with variance 1. Under the assumption of independent assignment, thus, the growth rates distribution would not change. But this is not our case. Indeed, if one follows the Polya process for the assignment of these 100 opportunities across the 100 firms, the growth rates density that emerges is different from a normal. This density is reported in Fig. 9 with the label $M = 100$ and is computed starting from the definition of the distribution in (11). As can be seen, the tails of the density are much fatter then in the normal case. This is a consequence of our mechanism of assignment: under the Polya process many opportunities tend to concentrate in few firms, producing final growth rates that are the sum of many micro-shocks and, consequently, can likely become quite large. The shape of the density is, however, still close to a normal, at least in the central part. But what happens if we further increase the number of opportunities? The answer is provided by the density shown in Fig. 9 with the label $M = 10000$. This is generated considering $M = 10000$ micro-shocks, i.e. an average of 100 micro-shocks per firms, distributed according to $N(x; 0, 1/100)$. Under the independent assignment hypothesis we would again obtain for the growth rates a normal density with unit variance. As can be seen, the obtained shape is instead almost identical to a tent-shaped Laplace distribution (see the inset of Fig. 9).

The agreement between the density generated by our assignment procedure and the Laplace can be further understood by looking at Fig. 10. Here we report the absolute deviation $|F_{\rm model}(x; M, N) - F_{\rm L}(x)|$ between the Laplace distribution and the distribution predicted by (11). We set the parameters equal to the same values used in Fig. 9. As can be seen, the absolute deviation is strongly reduced when the number of opportunities M is increased. On the other hand its value seems to depend on x. In order to build a global measure of agreement between the two distributions that is independent from the value of x we consider the height of the maximum of the functions plotted in Fig. 10. We define

$$D(N, \lambda) = max_{-\infty<x<+\infty} |F_{\rm model}(x; N, M) - F_{\rm L}(x)| \qquad (12)$$

where $F_{model}(x; N, M, f)$ is derived from the density defined in (11) and using normally distributed micro-shocks as described above, while $F_L(x)$ stands for the unit-variance Laplace distribution. The values of D for different N and M/N are plotted in Fig. 11. As N and M/N increase, the value of D decreases of several orders of magnitude and the ability of our mechanism to reproduce the Laplace distribution quickly improves. This picture tells us where, in the parameter space, we can expect that our mechanism gives a good account of the observed tent-like shape: this happens when both the number of firms N and the number of shocks per firm M/N are large. At this point a natural question arises: how much large should this "large" be? Of course the are no definite answers to this question, since for any finite value of N and M the maximum absolute deviation of F_{model} from F_{L} is not zero. Indeed, in the next Section we will show that the perfect agreement can be reached only for asymptotically large values of these parameters. However, a quite satisfactory even if "heuristic" answer[5] is provided by Fig. 12. Here, setting the value of N to "typical" values observed in data and choosing for M a value of a plausible order of magnitude, we obtain the same level of agreement to the exponential shape found in empirical investigations. Notice that the binned density in Fig. 12 is computed using 7 independent realizations of the assignment process to provide direct comparability to the empirical plots in Section 2.

We can conclude that the proposed mechanism of opportunities assignment based on Polya process does show a very good agreement with the shape of the one-yer growth rates density observed in industrial data. Before ending this Section let us briefly discuss the second evidence highlighted in Section 2, namely the progressive "normalization" of the growth rates density when one considers longer time scales (see Fig. 5). If one assumes that the process of opportunities assignment is repeated anew each year, i.e. that no memory of the previous year assignment is retained when the new year opportunities are assigned, then the growth rates of each year are independent for any firm and their T lags distribution tends toward a Gaussian when T becomes large. In this way we recover, at least as a first approximation, the behavior reported in Fig. 6. One can however argue that the idea of introducing strong positive-feedback effects in the opportunities assignment inside the same year and no memory at all of the previous year assignment sound rather inconsistent. After all, if dynamic increasing returns are there, why they should disappear during the new year's eve? We believe that the relevant point to notice here is that the one-year time span used to build empirical databases does not posses any meaning inside our model and, most probably, even inside the real economic dynamics (c.f. the discussion in Geroski (2000)). In this respect, one can think that the assignment procedure of our model works on a certain time span, let say on a time scale from 6 to 36 month, but that for longer time period the effect of the past captured opportunities fades away. This reduction in the relevance of opportunities caught in the far past can be progressive and smooth. From the behavior shown in Fig. 6, we can suppose that the reduction becomes relevant on a time scale of few years and, plausibly, acts with a different strength in different sectors. In order to describe this kind of dynamics one can probably modify the assignment mechanism introduce in Section 3 assuming, for instance, that the balls of a given firm are removed from the urn after a given time span or that their contribution to the probability of capturing new balls is inversely proportional to their "age", i.e. the number of turns they stayed in the urn. This kind of model, however, takes explicitly in consideration the flow of time and would introduce quite a few technical difficulties. We do not want to pursue here this issue but we think that this short digression provides some hints on how our model with "short-time dynamic increasing returns" should be interpreted.

[5]This point is further discussed in Bottazzi and Secchi (2002)

5 Asymptotic properties

In the previous Section we have shown that if N and M are set to proper (large) values and the micro-shocks are normally distributed then our model is able to reproduce the Laplace distribution.

We present now an analytical result proving that, as long as the total number of firms is large and their growth is generated by the assignment of a large number of small shocks, the final shape of the distribution of growth rates is robust to different specifications of the micro-shocks distribution. To this end, we study the model in the limit $M, N \to \infty$, $v_0 \to 0$ keeping $v = v_0\, M/N$ constant.

First of all it is convenient to express the relation in (11) in terms of densities

$$f_{\text{model}}(x; N, M, v_0) = \sum_{h=0}^{M} P(h; N, M)\; f(x; v_0)^{\star(h+1)} \tag{13}$$

Since we are finally interested in characterizing the growth rate distribution with random variables possessing well behaving probability densities the assumption that also micro-shocks posses densities $f(x; v_0)$ does not constitute a relevant reduction in generality. Before attacking the problem of the asymptotic behavior of (13) we need a simple preliminary result.

Lemma

The limit of the Bose-Einstein distribution defined in (9) when $M, N \to \infty$ with constant $\lambda = M/N$ is a Geometric distribution with mean λ.

Proof

Consider the generic term in (9), expanding the binomial coefficient one obtains

$$P(h; N, M) = \frac{(N + M - h - 2)!}{(N - 2)!\,(M - h)!}\,\frac{(N - 1)!\,M!}{(N + M - 1)!} \tag{14}$$

that after the expansion of the factorial terms and some easy simplifications reduces to

$$P(h; N, M) = \frac{(N - 1)\, M\,(M - 1)\, \ldots\, (M - h + 1)}{(N + M - 1)\,(N + M - 2)\, \ldots\, (N + M - h - 1)} \tag{15}$$

Dividing both numerator and denominator by N^{h+1} the expression can be rewritten as

$$P(h; N, M) = \frac{N - 1}{N}\, \frac{\frac{M}{N}\left(\frac{M}{N} - \frac{1}{N}\right) \ldots \left(\frac{M}{N} - \frac{h-1}{N}\right)}{\left(1 + \frac{M}{N} - \frac{1}{N}\right)\left(1 + \frac{M}{N} - \frac{2}{N}\right) \ldots \left(1 + \frac{M}{N} - \frac{h+1}{N}\right)}. \tag{16}$$

Substituting $M/N = \lambda$ and taking the limit for $N \to \infty$, the previous equation reduces to the h-th term of a geometric distribution

$$p_h(\lambda) = \frac{\lambda^h}{(1 + \lambda)^{h+1}}. \tag{17}$$

Q.E.D.

In the rest of this section we study the asymptotic properties of (13) when M and N become very large. Notice that when the number of opportunities M is increased the variance of the micro-shock distribution must be accordingly decreased. The idea is similar to the procedure adopted in the previous Section when the dependence of the growth rate density on the parameter M was analyzed. If one increases the average number of micro-shocks assigned to firms, then the variance of this shock must be decreased in order to maintain the same variance for the final distribution. When the limit is considered, the rescaling of the variance becomes mandatory to avoid degenerate distributions.

In order to easily perform the rescaling of the micro-shocks we assume the following functional form for the micro-shocks density

$$f(x;v_0) = \frac{1}{\sqrt{v_0}} m(\frac{x}{\sqrt{v_0}}) \quad . \tag{18}$$

where $m(x)$ is a density with unit variance. Again this form is introduced to facilitate the following analysis but does not reduce the generality of the results. Now, since the variance of the final density is $v = v_0 M/N$ and $\lambda = M/N$ we can set $v_0 = v/\lambda$ and using (18) the expression for the growth rates distribution in (13) becomes

$$f_{\rm model}(x;\lambda,v,N) = \sum_{h=0}^{\lambda N} P(h;N,\lambda N)\ (\sqrt{\frac{\lambda}{v}} m(\sqrt{\frac{\lambda}{v}}x))^{\star(h+1)}. \tag{19}$$

The variance of $F_{\rm model}$ in the previous equation does not depend on the value of N or λ.

Theorem

Suppose that the density m in (18) possesses the first two central moments. Then the summation in (19) converges in the limit for $N, \lambda \to \infty$ to a Laplace distribution with parameter $\sqrt{v/2}$, i.e.

$$\lim_{\lambda,N\to\infty} f_{\rm model}(x;\lambda,v,N) = f_L(x;\sqrt{v/2}) = \frac{1}{\sqrt{2v}}\ e^{-\sqrt{2/v}\ |x|} \tag{20}$$

Proof

Let us start by considering the characteristic function of the growth rates distribution

$$\tilde{f}_{\rm model}(k;\lambda,v,N) = \int_{-\infty}^{+\infty} dx\ e^{i\,k\,x}\ f_{\rm model}(x;\lambda,v,N) \tag{21}$$

The series in (19) is absolutely convergent and one can pass the Fourier integral inside the series to obtain

$$\tilde{f}_{\rm model}(k;\lambda,v,N) = \sum_{h=0}^{\lambda N} P(h;N,\lambda N)\ \tilde{m}(\sqrt{\frac{v}{\lambda}}k)^{(h+1)} \tag{22}$$

where $\tilde{m}$ is the characteristic function of the micro-shocks density $m(x)$ and we used the fact that the characteristic function of the h-convolution of density m is h times its characteristic function.

For any value of $\lambda > 0$ one can evaluate the limit for $N \to \infty$ and use the previous Lemma to obtain:

$$\tilde{f}_{\rm model}(k;\lambda,v) = \lim_{N\to\infty} \tilde{f}_{\rm model}(k;\lambda,v,N) = \frac{1}{1+\lambda} \sum_{h=0}^{\infty} \left(\frac{\lambda}{1+\lambda}\right)^h\ \tilde{m}(\sqrt{\frac{v}{\lambda}}k)^{(h+1)}. \tag{23}$$

Since $\tilde{m}$ is a characteristic function it is $|\tilde{m}(x)| \leq 1, \forall x$ (Lemma 1, Feller (1968) p.499), so that the geometric series in (23) can be resummed to give:

$$\tilde{f}_{\text{model}}(k; \lambda, v) = \frac{\tilde{m}(\sqrt{\frac{v}{\lambda}}\, k)}{1 + \lambda - \lambda\, \tilde{m}(\sqrt{\frac{v}{\lambda}}\, k)}. \tag{24}$$

Since the micro-shock distribution $m(x)$ possesses unitary second moment it admits the following expansion around the origin (Lemma 2, Feller (1971) p. 512)

$$\tilde{m}(\sqrt{\frac{v}{\lambda}}\, k) = 1 - \frac{1}{2}\,\frac{v}{\lambda}\, k^2 + o(\frac{1}{\lambda}) \tag{25}$$

Substituting this expansion in (24) and keeping only the leading term in $1/\lambda$ we obtain:

$$\tilde{f}_{\text{model}}(k; \lambda, v) = \frac{1 - \frac{1}{2}\,\frac{v}{\lambda} + o(1/\lambda)}{1 + \frac{v}{2}k^2 + o(1/\lambda)} \stackrel{\lambda \to +\infty}{\longrightarrow} \tilde{f}_{\text{model}}(k; v) = \frac{1}{1 + \frac{v}{2}k^2} \tag{26}$$

Since the limit function in (26) is continuous at the origin we can conclude that the original series in (19) converge to a proper density function (Theorem 2, Feller (1971) p. 508)[6].

Finally, substituting the definition of the Laplace density (1) in (21) and performing the integral it is easy to check that the last expression is the characteristic function of a Laplace distribution with parameter $a = \sqrt{v/2}$.

Q.E.D.

This theorem ensures that, when the number of firms and opportunities goes to infinity, the growth rates distribution generated by the model converges to the Laplace. Hence neither the fine tuning of the M and N parameters nor the choice of a particular micro-shocks distribution are required for our model to reproduce the observed tent-shape in the growth rates distribution.

6 Conclusions

In the present paper we add crucial evidence in support of the tent-shape of the firm growth rates distribution, extending previous findings with respect to two different directions. First we replicate the analysis already performed by Stanley et al. (1996) and Bottazzi et al. (2001) respectively on COMPUSTAT and PHID databases, on a new databank (MICRO.1) covering many firms in the Italian manufacturing industry. Second, using data disaggregated by sector, we prove that the shape of these distributions is not a mere effect of aggregation. Although intersectoral differences clearly arise, we conclude, in line with previous studies, that the tent-shape distribution of corporate growth rates appears as an extremely robust feature of the manufacturing industry, characterized by an higher regularity than the one shown by size distributions.

On the theoretical side, we present a model that describes the growth dynamics of firms. The model clearly originates in the Simon inspired literature on firm dynamics with which it shares two central features. First, different firms are viewed as different realizations of the same stochastic process. Second, the model includes a very simple idea of competition represented by a global constraint on the total number of available growth opportunities.

[6] The requirement of continuity in 0 is needed in order to assure that the limit density function is not defective.

The essential novelty of our approach lies in the use of a different mechanism for the assignment of these opportunities. The overall effect can be described as the emergence of a sort of "attracting force" between the various opportunities that tends to group them in bigger chunks leading to the appearance of two noticeable properties in their unconditional distribution: the presence of a fat tail, which indicates a more likely presence of extremely large number of opportunities assigned to a single firm and the absence of a natural scale of the underlying process, hinted by the 0 value of its mode.

The ability of the model to reproduce empirical findings without requiring a fine tuning of the parameters is ensured by the Theorem in Section 5 and constitutes its main strength. The search for an explanation of the emergence of tent-shape in the growth rates distribution in a framework where firms growth is interpreted as a cumulation of many small effects forcefully leads to the modified version of the Central Limit Theorem presented in this paper.

We are aware that the Polya urn mechanism presented in Section 3 does only constitute a simple metaphor. It can be taken, however, as the point of departure for further explanatory lines of thoughts, and constitutes the core of richer and more structured models of industrial dynamics. Indeed, due to the generality of the assumptions, our model can be used to account for the variegated sources of "positive feedback" dynamics, from increasing returns to network externalities, that strongly contribute in shaping the economic activity in the different sectors.

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Ateco code	Sector	# of Firms	Autocor.	Error	Estim. b	Estim. a
151	Production, processing and preserving of meat	114	-0.15	± 0.08	0.76	0.09
155	Dairy products	85	-0.17	± 0.09	0.78	0.07
158	Production of other foodstuffs (brad, sugar, etc...)	157	-0.11	± 0.07	0.95	0.10
159	Production of beverages (alcoholic and not)	94	0.21	± 0.08	0.84	0.11
171	Preparation and spinning of textiles	154	0.02	± 0.07	1.22	0.14
172	Textiles weaving	171	-0.01	± 0.06	1.12	0.12
173	Finishing of textiles	181	0.13	± 0.06	1.15	0.11
175	Carpets, rugs and other textiles	90	-0.13	± 0.09	1.04	0.11
177	Knitted and crocheted articles	162	-0.09	± 0.07	0.92	0.12
182	Wearing apparel	379	0.01	± 0.05	1.03	0.13
191	Tanning and dressing of leather	87	0.04	± 0.09	1.09	0.13
193	Footwear	245	-0.06	± 0.05	1.20	0.15
202	Production of plywood and panels	52	-0.09	± 0.11	0.90	0.10
203	Wood products for construction	59	-0.28	± 0.11	0.88	0.10
205	Production of other wood products (cork, straw, etc...)	56	0.18	± 0.11	1.33	0.10
211	Pulp, paper and paperboard	46	-0.37	± 0.12	0.93	0.11
212	Articles of paper and paperboard	180	-0.19	± 0.06	1.01	0.11
221	Publishing	72	-0.11	± 0.10	0.78	0.08
222	Printing	199	-0.03	± 0.06	1.26	0.11
241	Production of basic chemicals	80	-0.17	± 0.09	0.91	0.11
243	Paints, varnishes, printing inks and mastics	58	-0.07	± 0.11	1.08	0.08
244	Pharmaceuticals, medicinal chemicals and botanical products	97	0.07	± 0.08	0.88	0.11
245	Soap and detergents, cleaning and toilet preparations	46	0.35	± 0.12	0.97	0.10
246	Other chemical products	51	-0.04	± 0.11	0.79	0.10
251	Rubber products	87	0.04	± 0.09	1.05	0.10
252	Plastic products	352	-0.12	± 0.04	1.01	0.12
261	Glass and glass products	87	-0.11	± 0.09	1.08	0.10
262	Ceramic goods not for construction	59	0.26	± 0.11	1.18	0.10
263	Ceramic goods for construction	91	0.09	± 0.08	1.04	0.11
264	Bricks, tiles and construction products in baked clay	84	-0.03	± 0.09	1.16	0.10
266	Articles in concrete, plaster and cement	141	-0.21	± 0.07	0.95	0.16
267	Cutting, shaping and finishing of stone	69	-0.03	± 0.10	1.16	0.11
273	First processing of iron and steel	82	0.05	± 0.09	0.77	0.12
275	Casting of metals	125	-0.13	± 0.07	1.04	0.12
281	Structural metal products	156	-0.18	± 0.07	1.31	0.19
284	Forging, pressing, stamping and roll forming of metal	132	-0.07	± 0.07	1.14	0.13
285	Treatment and coating of metals	182	-0.13	± 0.06	1.02	0.14
286	Cutlery, tools and general hardware	149	-0.07	± 0.07	0.98	0.12
287	Other fabricated metal products	265	-0.18	± 0.05	0.99	0.11
291	Machinery for the production and the use of mechanical power	224	-0.02	± 0.05	1.00	0.12
292	Other general purpose machinery	199	-0.22	± 0.06	1.04	0.16
293	Agricultural and forestry machinery	54	-0.34	± 0.11	0.97	0.14
294	Machine tools	114	-0.11	± 0.08	1.08	0.17
295	Other special purpose machinery	424	-0.24	± 0.04	1.08	0.18
297	Domestic appliances not elsewhere classified	59	-0.07	± 0.11	1.19	0.11
311	Electric motors, generators and transformers	71	-0.01	± 0.10	0.94	0.13
312	Manufacture of electricity distribution and control equipment	70	-0.16	± 0.10	0.78	0.14
316	Electrical equipment not elsewhere classified	91	-0.11	± 0.09	1.02	0.14
322	TV and radio transmitters and lines for telephony and telegraphy	44	-0.15	± 0.12	0.92	0.17
332	Measure, control and navigation instruments	51	0.09	± 0.11	1.19	0.12
342	Production of bodies for cars, trailers and semitrailers	50	-0.07	± 0.11	1.10	0.15
343	Production of spare parts and accessories for cars	125	-0.09	± 0.07	1.05	0.13
361	Furniture	444	-0.02	± 0.04	0.97	0.12
362	Jewelry and related articles	84	0.05	± 0.09	1.36	0.16
366	Miscellaneous manufacturing not elsewhere classified	68	0.10	± 0.10	1.16	0.12

Table 1: Summary table of the 55 sectors under analysis. The estimated a and b parameters together with the average autocorrelation coefficient are reported.