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Abstract
An urn-ball probabilistic model of the labour market is developed. Agents can be employed, (voluntary or involuntary) unemployed or entrepreneurs. The analytical long run equilibrium probabilities for each state and the matching function are derived. Then, the out-of-equilibrium dynamics are investigated through an agent-based simulation, which provides also results on firm demography. The simulation model is finally used to investigate departures from maximizing individual behaviour and the effects of more realistic assumptions about profits and the business cycle.

Introduction

In the economic literature, search models have become the standard reference for the analysis of unemployment. They originated from Stigler (1961) work on the economics of information, who considered a buyer choosing the number of price quotations before beginning the search process, in order to minimize expected price plus sampling cost. Search models have been first applied to labour issues, in a more dynamic perspective, with the work of Phelps et al. (1970). A surge in this strand of the literature occurred during the eighties, with major contributions by Diamond (1981, 1982a,b), Mortensen (1982a,b), and Pissarides (1984a,b). Mortensen and Pissarides (1999) and Pissarides (2000a,b) themselves provide extensive reviews of search models for the labour market.

With respect to the standard competitive equilibrium theory, in search models trade is explicitly affected by the dynamics of the process, and by the related uncertainty. «One thing competitive theory gets right with supply and demand is that prices are determined endogenously. Then again, one thing it gets wrong is that they are not chosen by anyone in the model, but by something outside the model – the auctioneer. The auctioneer is a very convenient device for solving or at least getting around the problem of price formation, but presumably this cannot be the last word on the problem. Search models not only allow us to discuss ways in which wages and other prices are determined by agents in the model, they allow us to study a wide variety of alternative ways of endogenizing prices, including bilateral bargaining, ex ante wage posting by employers, and other mechanisms» (Rogerson and Wright, 2002). Thus, to a certain extent, search theory goes in the direction auspicated by ACE practitioners, as described in Richiardi (2003). Moreover search theory, with its focus on the analysis of *frictions,* has found many interesting applications also outside the field of labour economics. Monetary economics (Kiyotaki and Wright, 1993; Shi, 1995; Trejos and Wright, 1995), industrial organization (Jovanovic, 1982; Jovanovic and Rob, 1989; Jovanovic and MacDonald, 1994), and family economics (Mortensen, 1988; Burdett and Coles, 1997, 1999; Shimer and Smith, 2000) have all benefited from this theoretical framework.

However, search models remain intrinsically equilibrium models, developed in the framework of optimising behaviour. Search models rely on three pillars: the decision of workers, the decision of firms and the wage setting mechanism. Search activity is costly for individuals, who compare the utility of possibly getting a job with their actual utility (which may come from unemployment benefits, or from their present wage if on-the-job search is allowed), thus computing the equivalent of an arbitrage equation for the valuation of an option (selling their unit of labour) in a perfect capital market.

When two-sided search is considered, instead of considering an exogenous number of vacancies as in simpler models, the search process by firms is explicitly modelled. Vacancy opening is costly for firms. They thus make optimising choices by comparing expected profits from posting a vacancy with those of leaving the growth opportunity unexploited. In a competitive economy these profits have to be null, allowing for simpler solutions.

Wage setting follows two approaches. The most common considers a Nash bargaining solution (Diamond, 1982), in order to split the rent generated by the search costs between the firm and the applicant. The second approach considers wage posting by the firms. Three different types of wage posting have been proposed. With a *single offer, private information* mechanism, workers are revealed the wage offer only upon (sequential) contact (Diamond, 1971). With a *competitive search* mechanism, firms publicly post their wage offers, workers apply to only one job at a time but they are not sure to get the job, should they apply, because it may have already gone to another applicant (Moen, 1997). Finally, with a *multiple offers, private information* mechanism workers are allowed to consider more than one wage offer at a time (as with on-the-job search), but still they get to know the offer only upon contact (Burdett and Judd, 1983).

A related problem with wage posting is determining how firms choose wage offers. Ideally, this should also be endogenous, and lead to a non-degenerate wage distribution, in equilibrium. Diamond (1971) was one of the first to address this issue. However, his model produces a single equilibrium wage, equal to the value of unemployment benefits, even if firms have different productivity. A number of models leading to non-degenerate wage distributions, both in the market for goods and in the market for labour, were subsequently developed. Firms offer higher wages in order to attract or retain more or better workers. A number of mechanisms, including on-the-job search and workers heterogeneity (with respect to the utility deriving from unemployment, or to productivity) can lead to such an outcome - see Rogerson and Wright (2002) for a detailed survey.

Ex-ante suitable jobs may include all stock of jobs (*urn-ball matching*) or, in case job seekers have complete information about available vacancies[1] may include only new jobs (*stock-flow matching*).

From an aggregate point of view, the search activity leads to a matching function (Blanchard and Diamond, 1990), which relates the number of matches with input variables like the unemployment rate and the number of vacancies (see Petrongolo and Pissarides, 2001). The analysis of labour markets through aggregate matching models is a stream of research on its own. While search models have demonstrated the existence of equilibria in a decentralized wage setting with frictions, starting from the analysis of the micro-foundations of such a market, the matching function «as well as recognizing the simultaneous occurrence of unemployment and vacancies»[2] has focused on the flows of hires, although from an aggregate point of view. Only recently the links between micro-founded search models and the aggregate matching function has been investigated. The matching function has an informational content similar to that of the production function in a production process. However, most attempts to derive it from its micro-foundations do not take into consideration the relationship fact that the number of vacancies also depends on unemployment, the two things being jointly determined and endogenous.

[1] and thus unmatched workers and vacancies will not match, even in future rounds
[2] Ballot (2002)

The search framework is so well established that many contributions have explored extension of the basic models. Shimer and Smith (2001), in a general search model that abstracts from the labour market, show that the decentralised equilibrium with heterogeneous agents is inefficient. Acemoglu (2000) and Albrecht and Vroman (2000) introduce skill heterogeneity among workers, and investigate the implications for wages and unemployment rate for each group. Search intensity has been endogenised as a choice variable (Pissarides, 2000). Albrecht et al. (2003) consider the implications of multiple applications by job seekers.

However, search models still ignore many features of real labour markets. In particular, firms hardly exist. They are replaced by vacancies, i.e. by single-job entities. Old firms never die; new firms never come to life: instead, jobs appear and disappear. Job creation is endogenous, but job destruction is generally exogenously given. A first attempt to provide a more realistic description of layoffs is found in Burdett and Mortensen (1980), where each job offer is characterised by two variables – a wage and a constant probability of the position being closed down. Clearly, this is still a very poor way of considering job destruction. In order to make advances, two mechanisms have been conceived. One considers (stochastic) shocks to the productivity of each job. The job is then closed down if its productivity falls beyond a minimum threshold (Mortensen and Pissarides, 1994). An alternative way involves considering job obsolescence over time. Old jobs offer smaller wages. Thus, they'll find increasingly hard to attract workers, and will eventually closed down (Aghion and Howitt, 1994; Caballero and Hammour, 1994). Note that in both cases job destruction does not depend on unemployment. Having only a very naïf description of firms, these models never allow job creation and job destruction to depend on variables such as the number of firms in the market, or the dimension of the firm. Moreover, job destruction is generally not modelled separately from firing decisions. This implies the impossibility of distinguishing between employee and job turnover. The only other way to include such a distinction without modelling layoffs is through on-the-job-search. The number of vacancies must then be updated accordingly: it must be higher the greater the number of job-to-job changes. Burdett (1978) provides a first attempt to model on-the-job search. However, in his model there is no determination of the number of vacancies. Thus, job quits may be indifferently interpreted as job destruction. On-the-job search intensity may depend on experienced wage shocks, or on learning about the utility deriving from that work (Jovanovic, 1979). In particular, since learning increases with tenure, models like Jovanovic's imply that workers with longer tenures are less likely to quit, and are more likely to be gaining higher wages.

Also, the realism of optimising behaviour could be questioned. There is a strong empirical literature showing that labour market choices are often made on the basis of rules of thumbs, which may be inefficient (Sargent, 1993; Leijonhufvud, 1993).

Overall, search models offer a coherent modelling framework. However, different models can be distinguished along many dimensions, while sharing common tools and methods. Most models, aimed at investigating a particular issue, only include some of the relevant features. Actually, due to the analytical difficulty involved, it is indeed very hard to think of a comprehensive search model, in which investigate for instance the effects of different individual and firm behaviour, or different institutional settings. This could be pursued in a simulation model. However, to the extent of my knowledge only few attempts have been made to replicate the working of a complete labour market. Ballot (2003) describes a simulation of the French labour market, with bounded rationality workers and firms, a rich specification of the search process on both sides for different sets of institutional rules and the presence of intermediaries. Interesting and comprehensive microsimulation models of the labour market have been developed, in the spirit of Orcutt (1957, 1961), but their focus is most often on the supply of labour and the interaction with demographic issues like ageing (see for instance Martini, 1997). Nagel (1998) has a search model where agents receive wage offers and perform a simple hill-climbing towards the company that pays the highest wage. However, firms can reject applicants because of bankruptcy, in which case workers start again from an unemployment state. The focus is on the implications of the model on inflation dynamics. Neugart (2003) focuses on recovering by means of an ACE model some empirical properties of the aggregate matching function.

The purpose of this paper is adding the analysis of firm dynamics in a search-theoretic framework. In order to remain more closely related to the existing literature, I provide a simple reference analytical model. Then, an agent-based simulation of the model is developed, in order to explore the out-of-equilibrium dynamics and the effects of some violations in the main assumptions. In particular, a more behaviourist version of the model, with agents following rules-of-thumb, will be presented. Finally, more structured hypothesis on the value of some relevant parameters governing the business cycle and the profitability of firms will be introduced. The model is set-up in section 1. State transition probabilities are derived in section 2. Section 3 characterizes the long-run equilibrium of the system. Section 4 presents an agent-based implementation of the model, and investigates firm dynamics. Section 5 deals with the above mentioned extensions of the model, while section 6 concludes.

1. The model

The model belongs to the class of urn-ball search models, with private information, single offer. However, a rather different modelling approach is considered. Optimal individual choice rules are outlined, given a two-step decision process where workers have inertia and change their job only when their satisfaction level falls below a threshold, irrespective of the utility deriving from other choices. Then, the a-priori probability of each choice being taken, in equilibrium, is computed. This allows filling a transition matrix, for each state of the system (unemployment, employment, self-employment), defining a regular Markov chain. The long-run probabilities for each state are then computed, using the global balance equations implied by the Markov chain. The approach is similar to that of Diermeier and Van Mieghem (2001).

1.1 Labour supply

Individuals can be self-employed, employed or unemployed. At every period they face the following four possible choices:

Table 1: Individual choices

Stay (*)	Remain in the present organization (firm)
Join	Apply for another job
Start	Found a new startup
Relax	Withdraw from the labor market

(*) only if currently employed

Thus, the state transition matrix looks like the following table:

Table 2: State transition matrix

<table>
<tr><td></td><td colspan="3">Ending state</td></tr>
<tr><td>Starting state</td><td>Unemployed</td><td>Employed</td><td>Self-employed</td></tr>
<tr><td>Unemployed</td><td>Unsuccessful Join
Relax</td><td>Successful Join</td><td>Start</td></tr>
<tr><td>Employed</td><td rowspan="2">Unsuccessful Stay
Unsuccessful Join
Relax</td><td rowspan="2">Successful Stay
Successful Join</td><td rowspan="2">Start</td></tr>
<tr><td>Self-employed</td></tr>
</table>

Each individual has a reservation wage, r (which may vary over time). Individuals are risk neutral. They first compute their expected wage in the present state, and compare it with their reservation wage. They try to change their status only if their expected wage is below r. Thus, they do not compute every time an expected wage for every possible decision. As many real people do, they have inertia, and prefer not to change, unless they are forced to. If this is the case, they decide either to look for a new job, or to become entrepreneurs, or to remain idle, by comparing the expected payoffs of the different choices. Once they decide to apply for other jobs, they quit their present firm, if employed. Should all applications fail, they thus fall into unemployment.

Letting w^e be the expected wage, choices are thus given by:

Table 3: Comparing expected utility

Employed	
Stay	$w^e_{STAY} \geq r$
Join	$w^e_{STAY} < r, w^e_{JOIN} \geq w^e_{START}, w^e_{JOIN} \geq r$
Start	$w^e_{STAY} < r, w^e_{START} > w^e_{JOIN}, w^e_{START} \geq r$
Relax	$\max(w^e_{STAY}, w^e_{JOIN}, w^e_{START}) < r$
Unemployed	
Join	$w^e_{JOIN} \geq w^e_{START}, w^e_{JOIN} \geq r$
Start	$w^e_{START} > w^e_{JOIN}, w^e_{START} \geq r$
Relax	$\max(w^e_{JOIN}, w^e_{START}) < r$

Let

[1]
$$\begin{aligned}
P_{STAY} &= \Pr(w^e_{STAY} \geq r) \\
P_{JOIN} &= \Pr(w^e_{JOIN} \geq w^e_{START}, w^e_{JOIN} \geq r) = \\
&= \Pr\left(w^e_{JOIN} \geq w^e_{START}\right) \cdot \Pr\left(w^e_{JOIN} \geq r \mid w^e_{JOIN} \geq w^e_{START}\right) \\
P_{START} &= \Pr(w^e_{START} > w^e_{JOIN}, w^e_{START} \geq r) = \\
&= \Pr\left(w^e_{START} > w^e_{JOIN}\right) \cdot \Pr\left(w^e_{START} \geq r \mid w^e_{START} > w^e_{JOIN}\right) \\
P_{RELAX} &= \Pr(\max(w^e_{JOIN}, w^e_{START}) < r) = \\
&= \Pr\left(r > w^e_{JOIN}\right) \cdot \Pr\left(w^e_{JOIN} \geq w^e_{START} \mid r > w^e_{JOIN}\right) + \\
&+ \Pr\left(r > w^e_{START}\right) \cdot \Pr\left(w^e_{START} \geq w^e_{JOIN} \mid r > w^e_{START}\right)
\end{aligned}$$

It is now possible to fill in the probabilities for each cell of the ‘a priori’ transition matrix:

Table 4: State transition matrix

	Ending state		
Starting state	**Unemployed**	**Employed**	**Self-employed**
Unemployed	$P_{JOIN} \cdot P_{JOIN}^{UNSUCC} + P_{RELAX}$	$P_{JOIN} \cdot P_{JOIN}^{SUCC}$	P_{START}
Employed / **Self-employed**	$P_{STAY} P_{STAY}^{UNSUCC} + $ $+ (1 - P_{STAY})(P_{JOIN} \cdot P_{JOIN}^{UNSUCC} + P_{RELAX})$	$P_{STAY} \cdot P_{STAY}^{SUCC} +$ $+ (1 - P_{STAY}) \cdot P_{JOIN} \cdot P_{JOIN}^{SUCC}$	$(1 - P_{STAY}) \cdot$ $\cdot P_{START}$

There is a fixed number of $N = N_{STAY} + N_{JOIN} + N_{START} + N_{RELAX}$ individuals. The expected number of workers willing to stay is $N_{STAY} = NeP_{STAY}$ and the expected number of applicants is $N_{JOIN} = N(u + e(1 - P_{STAY}))P_{JOIN} = N(1 - eP_{STAY})P_{JOIN}$. Finally, the expected number of new start-ups is $N_{START} = Ne(1 - P_{STAY})P_{START} + NuP_{START} = N(1 - eP_{STAY})P_{START}$. In addition, there is a (variable) number of F_t firms. Individuals and firms are not located in space: every worker can contact any firm.

1.2 Labor demand

Each individual has, each period, a business idea, whose exploitation requires a new startup and J_i units of labor, with J_i randomly extracted from a distribution D_J. These business opportunities are valid only for one period. Once a firm is set up, job opportunities grow at a rate g_t with g_t randomly extracted each period for all firms from

be purely stochastic. This means that a W_f employees firm at time t will try to become a $W_f \cdot (1 + g_t)$ employees firm at time t + 1, thus opening (or destroying) $W_f \cdot g_{,t}$ positions. The number of available vacancies will be equal to the number of new positions, plus the number of old positions left vacant by employees that have decided to leave the firm. Workers make their decisions before the realization g_t is revealed. Note that a positive realization of g_t does not automatically imply a particular firm or the economy as a whole will expand, since vacancies could remain unfilled.

1.3 Wages

All firms in the market are able to get every period a market return of $W_f \cdot (1 + s_{f,t})$, where $(1 + s_{f,t})$ is an a priori unknown firm- and time-specific multiplier, with $s_{f,t}$ randomly extracted from a distribution D_s. Start-ups bring an additional cost of αJ_i for the entrepreneur. This cost is proportional to the size of the business opportunity, and accounts for all kind of set-up costs. After the first period, all differences between employer and employees disappear.

The wage shock $s_{f,t+1}$ becomes known to employees before they take their decision about whether to leave the firm, but it is not known to applicants. Here, $s_{f,t}$ accounts both for monetary and non-monetary rewards, which could well be assumed to be an experience good. All employees are equally rewarded. Wages are thus equal to $(1 + s_{f,t})$. Of course it would be reasonable to think of $s_{f,t}$ as being correlated over time, or across firms, or to be somehow related to the business cycle parameter g_t. Section 5 will put some more structure on this parameter. Here, for the sake of simplicity, it is considered to be purely idiosyncratic.

Thus, each employee receives $w_{f,t} = (1 + s_{f,t})$, while the founder receives $(1 + s_{f,t}) - \alpha J_i$. Workers are aware of the uncertainty over s in the aggregate. Consequently, their expectations are:

[2]
$$
\begin{aligned}
w^e_{STAY,f} &= \left(1 + s_{f,t+1}\right) \cdot P^{SUCC}_{STAY} \\
w^e_{JOIN} &= \left(1 + \bar{s}\right) \cdot P^{SUCC}_{JOIN} \\
w^e_{START} &= \left(1 + \bar{s}\right) - \alpha \cdot J_i
\end{aligned}
$$

Since s_f is independent of s_j, for any f and j, knowing that $w^e{}_{STAY}$ is smaller than r has no influence on the expected values $w^e{}_{JOIN}$ and $w^e{}_{START}$. Thus, we can write, for employed workers:

$$[3] \quad P^{E}_{JOIN} = \Pr(w^{e}_{STAY} < r, w^{e}_{JOIN} \geq w^{e}_{START}, w^{e}_{JOIN} \geq r) = (1 - P_{STAY}) \cdot P_{JOIN}$$
$$P^{E}_{START} = \Pr(w^{e}_{START} < r, w^{e}_{START} > w^{e}_{JOIN}, w^{e}_{START} \geq r) = (1 - P_{STAY}) \cdot P_{START}$$
$$P^{E}_{RELAX} = \Pr(\max(w^{e}_{STAY}, w^{e}_{JOIN}, w^{e}_{START}) < r) = (1 - P_{STAY}) \cdot P_{RELAX}$$

1.4 Stay

As explained above, at every period firms decide how many jobs they can sustain. Jobs are first given to old employees, by means of a tournament. Only in case the number of jobs exceeds the number of employees willing to stay, new vacancies are opened.

If $N_{STAY,f}$ and J_f are respectively the number of employees willing to stay and the number of job opportunities available at firm *f*, the probability of being confirmed, once a Stay decision is taken, is thus $\min\left(\frac{J_f}{N_{STAY,f}}, 1\right)$. The *a priori* probability of a successful Stay, given the worker is employed and has decided to stay, is:

$$[4] \quad P^{SUCC}_{STAY} = \Pr\left(J_f \geq N_{STAY,f}\right) + \Pr\left(J_f < N_{STAY,f}\right) \cdot E\left(\frac{J_f}{N_{STAY,f}} \middle| J_f < N_{STAY,f}\right)$$

In the simplest case with no heterogeneity among workers (i.e. $r_i = r$), all workers in the same firm take the same decision, regarding whether to stay or not.

If they all decide to stay, the probability of being confirmed depends on the business cycle parameter g_t. For positive realizations of g_t this probability is 1, while for negative realizations this probability is 1+ $E[g_t|\ g_t < 0]$. Suppose g is uniformly distributed between $g_L \in (-1,0)$ and $g_H > 0$, then:

$$[5] \quad P^{SUCC}_{STAY} = \frac{g_H}{g_H - g_L} - \frac{g_L}{g_H - g_L} \cdot \left(1 + \frac{g_L}{2}\right) = 1 - \frac{g_L^2}{2(g_H - g_L)}$$

and

$$P^{UNSUCC}_{STAY} = \frac{g_L^2}{2(g_H - g_L)}$$

1.5 Join

Workers apply to vacancies, not to firms, as in standard search models. Each worker has a fixed number of *A* applications to send, in case he decides to look for a new job. Vacancies collect all their applications (if any) and select randomly a prospective worker. The worker accepts the first offer he receives. He discovers the firm specific wage only after being hired.

The total number of vacancies in existing firms is:

[6] $$V_F = \sum_{f=1}^{F} \max(J_f - N_{STAY,f}, 0)$$

With homogenous workers, all employees take the same decision, regarding whether to stay or not. Thus, the expected number of vacancies in any one firm, given no information about $s_{f,t+1}$ is publicly available, is given by:

[7] $g_{t+1} \leq 0$:

$$V^e_{f,t+1} = \begin{cases} W_{f,t} \cdot (1 + g_{t+1}) & \text{with prob.} \quad 1 - P_{STAY} \\ 0 & \text{with prob.} \quad P_{STAY} \end{cases}$$

$g_{t+1} \geq 0$:

$$V^e_{f,t+1} = \begin{cases} W_{f,t} \cdot (1 + g_{t+1}) & \text{with prob.} \quad 1 - P_{STAY} \\ W_{f,t} g_{t+1} & \text{with prob.} \quad P_{STAY} \end{cases}$$

Hence:

[8] $$V^e{}_{f,t+1} = W_{f,t}(1 + E(g_{t+1}))(1 - P_{STAY}) + W_{f,t} E(g_{t+1} | g_{t+1} \geq 0) \frac{g_H P_{STAY}}{g_H - g_L}$$
$$= W_{f,t} \cdot \frac{2(g_H - g_L)(1 - P_{STAY}) + (g_H^2 - g_L^2(1 - P_{STAY}))}{2(g_H - g_L)}$$

and the total expected number of vacancies in existing firms is:

[9] $$V^e_F = eN \cdot \frac{2(g_H - g_L)(1 - P_{STAY}) + (g_H^2 - g_L^2(1 - P_{STAY}))}{2(g_H - g_L)}$$

Note again that the expected value of g is not affected by the fact that the worker will consider the option to apply for another job only after having decided to quit his present firm, since individual wage shocks are uncorrelated.

The expected number of vacancies in new start-ups is:

[10] $$V_S^e = N(1 - eP_{STAY})P_{START}\bar{J}$$

and the total expected number of vacancies is $V^e = V_F^e + V_S^e$.

Following Albrecht et al. (2003), the probability that any one applicant has applied to a particular vacancy is A/V^e, so the number of application to a particular vacancy is $N_v = \mathrm{bin}(N_{JOIN}, A/V^e)$.

The probability that the vacancy has at least an application to consider, assuming $V^e \geq A$, is:

[11] $$p = 1 - \left(1 - A / V^e\right)^{N_{JOIN}}$$

From the perspective of the individual, when one application is sent out, the probability that it is selected is 1 over the number of applications received for that vacancy. On average, this number is equal to the number of applications sent out (AN^{JOIN}) over the number of vacancies that receive applications (pV^e).
Consequently, the probability an application is selected for a vacancy, given $r_i = r$, is $q = \dfrac{p \cdot V^e}{A \cdot N_{JOIN}}$. Note that in considering what happens to a particular vacancy we are considering the case $N_{JOIN} \geq 1$. The probability of being selected for at least one vacancy is $1 - (1 - q)^A$. Therefore, the a priori probability of a successful Join, given a Join decision, is:

[12] $$P_{JOIN}^{SUCC} = 1 - \left(1 - \frac{p \cdot V^e}{A \cdot N_{JOIN}}\right)^A$$

and the a priori probability of an unsuccessful Join is $P_{JOIN}^{UNSUCC} = \left(1 - \frac{p \cdot V^e}{A \cdot N_{JOIN}}\right)^A$

At this point, letting *e* and *u* be respectively the employment and unemployment rate, it is also possible to write down the matching function:

[13] $$\begin{aligned} & M(u,e,V,A) = M(N,g,s,J,r,\alpha,A) = \\ & = N_{JOIN} \cdot P_{JOIN} \cdot P_{JOIN}^{SUCC} = N \cdot P_{JOIN} \cdot P_{JOIN}^{SUCC} \cdot (1 - eP_{STAY}) \end{aligned}$$

1.6 Start

In order to successfully form a start-up, no particular requirements are necessary, and at least one vacancy is automatically filled (the founder). Since the recruiting mechanism involves first choosing one applicant, and then asking if he is willing to join at the prospected wage, the probability that the selected applicant has not been recruited yet for other vacancies is proportional to the number of the selected worker's applications receiving positive answers, $(A\text{-}1) \cdot q + 1$. The probability of filling any one vacancy is thus:

[14] $$z = \frac{p}{(A-1) \cdot q + 1}$$

Hence, the average number of vacancies a J_i startup will be able to fill is:

[15] $$W_i^e = (J_i - 1) \cdot z + 1$$

2. Choices

Supposing s is uniformly distributed between $s_L \in$ (-1,0) and s_H >0. Substituting into equation [1] yields:

$$[16] \quad P_{STAY} = \Pr(w_{STAY}^{e} \geq r) = \Pr\left(s_f \geq \frac{2(g_H - g_L)(r-1) + g_L^2}{2(g_H - g_L) - g_L^2} \right) =$$

$$= \begin{cases} 0 & \text{for } a > s_H \\ \dfrac{s_H - a}{s_H - s_L} & \text{for } a \in [s_L, s_H]; \\ 1 & \text{for } a < s_L \end{cases}$$

$$a = \frac{2(g_H - g_L)(r-1) + g_L^2}{2(g_H - g_L) - g_L^2}$$

Note that the lower threshold for a, when r = 0, is a = -1

Now, suppose J is uniformly distributed between $J_L \geq 0$ and J_H. In order to obtain P_{JOIN}, P_{START} and P_{RELAX} we must distinguish between the following cases (see the Appendix):

$$(a) \quad \frac{1+\bar{s}-r}{\alpha} \geq J_H$$

$$(a.1) \quad P_{JOIN}^{SUCC} \leq \frac{r}{1+\bar{s}}$$

$$(a.2) \quad P_{JOIN}^{SUCC} \in \left[\frac{r}{1+\bar{s}}, \frac{1+\bar{s}-\alpha J_H}{1+\bar{s}} \right]$$

$$(a.3) \quad P_{JOIN}^{SUCC} \in \left[\frac{1+\bar{s}-\alpha J_H}{1+\bar{s}}, \frac{1+\bar{s}-\alpha J_L}{1+\bar{s}} \right]$$

$$(a.4) \quad P_{JOIN}^{SUCC} \geq \frac{1+\bar{s}-\alpha J_L}{1+\bar{s}}$$

(b) $\quad \dfrac{1+\bar{s}-r}{\alpha} \in [J_L, J_H]$

$(b.1)$ $\quad P_{JOIN}^{SUCC} \le \dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}$

$(b.2)$ $\quad P_{JOIN}^{SUCC} \in \left[\dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}, \dfrac{r}{1+\bar{s}}\right]$

$(b.3)$ $\quad P_{JOIN}^{SUCC} \in \left[\dfrac{r}{1+\bar{s}}, \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}\right]$

$(b.4)$ $\quad P_{JOIN}^{SUCC} \ge \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}$

(c) $\quad \dfrac{1+\bar{s}-r}{\alpha} \le J_L$

$(c.1)$ $\quad P_{JOIN}^{SUCC} \le \dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}$

$(c.2)$ $\quad P_{JOIN}^{SUCC} \in \left[\dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}, \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}\right]$

$(c.3)$ $\quad P_{JOIN}^{SUCC} \in \left[\dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}, \dfrac{r}{1+\bar{s}}\right]$

$(c.4)$ $\quad P_{JOIN}^{SUCC} \ge \dfrac{r}{1+\bar{s}}$

We then obtain the following results:

Table 5: Choices probabilities

Case		P_{JOIN}	P_{START}	P_{RELAX}	sum
I	a.1	0	1	0	1
I	a.2	0	1	0	1
II	a.3	$\dfrac{\alpha J_H-(1+\bar{s})\left(1-P_{JOIN}^{SUCC}\right)}{\alpha(J_H-J_L)}$	$\dfrac{(1+\bar{s})\left(1-P_{JOIN}^{SUCC}\right)-\alpha J_L}{\alpha(J_H-J_L)}$	0	1
III	a.4	1	0	0	1
IV	b.1	0	$\dfrac{1+\bar{s}-r-\alpha J_L}{\alpha(J_H-J_L)}$	$\dfrac{\alpha J_H-(1+\bar{s}-r)}{\alpha(J_H-J_L)}$	1
IV	b.2	0	$\dfrac{1+\bar{s}-r-\alpha J_L}{\alpha(J_H-J_L)}$	$\dfrac{\alpha J_H-(1+\bar{s}-r)}{\alpha(J_H-J_L)}$	1
II	b.3	$\dfrac{\alpha J_H-(1+\bar{s})\left(1-P_{JOIN}^{SUCC}\right)}{\alpha(J_H-J_L)}$	$\dfrac{(1+\bar{s})\left(1-P_{JOIN}^{SUCC}\right)-\alpha J_L}{\alpha(J_H-J_L)}$	0	1
III	b.4	1	0	0	1
V	c.1	0	0	1	1
V	c.2	0	0	1	1
V	c.3	0	0	1	1
III	c.4	1	0	0	1

different cases leading to the same probabilities have been grouped and labelled I to V.

3. Long run equilibrium

Now, **case I** being characterized by P_{JOIN} = [illegible] getting a vacancy, conditional on applying for it, is equal to 1 [illegible] us impossible that $P_{JOIN}^{SUCC}<\dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}\leq 1$. The same is true for **case IV**, where it is impossible that $P_{JOIN}^{SUCC}<\dfrac{r}{1+\bar{s}}\leq\dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}\leq 1$.

Therefore the table above simplifies to:

Table 6: Choices probabilities

Case	P_{JOIN}	P_{START}	P_{RELAX}	Sum
II	$\dfrac{\alpha J_H - (1+\bar{s})\left(1-P_{JOIN}^{SUCC}\right)}{\alpha(J_H - J_L)}$	$\dfrac{(1+\bar{s})\left(1-P_{JOIN}^{SUCC}\right)-\alpha J_L}{\alpha(J_H - J_L)}$	0	1
III	1	0	0	1
V	0	0	1	1

Case II corresponds to cases a.3 and b.3 of table 5.
Case III corresponds to cases a,b,c.4 of table 5
Case V corresponds to cases c.1,2,3 of table 5

Note that when $J_L = 0$, $\dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}} = 1$, and case III becomes very unlikely. Note also that the reservation wage does not directly affect individual choices, once a leave decision is taken.

In **case V** the transition matrix looks like:

Table 7: Case V transition matrix

	Ending state		
Starting state	**Unemployed**	**Employed**	**Self-employed**
Unemployed	1	0	0
Employed / **Self-employed**	$1-P_{STAY}P_{STAY}^{SUCC}$	$P_{STAY}\cdot P_{STAY}^{SUCC}$	0

with unemployment being the absorbing state. Note that case V implies $P_{JOIN}^{SUCC} = 0$, which is coherent with its premises.

More generally, the transition matrix of table 4 defines a regular Markov chain with stationary transition probabilities. Its limiting distribution, i.e. the long run probability to find the process in each state, irrespective of the initial state (which is also the long run mean fraction of time that the process is in each state) is given by:

[17] $$\pi_e = \frac{P_{JOIN} P_{JOIN}^{SUCC} + P_{STAY} P_{STAY}^{SUCC} P_{START}}{1 + P_{STAY}\left(P_{START} + P_{JOIN} P_{JOIN}^{SUCC}\right) - P_{STAY} P_{STAY}^{SUCC}}$$

$$\pi_s = \frac{P_{START}\left(1 - P_{STAY} P_{STAY}^{SUCC}\right)}{1 + P_{STAY}\left(P_{START} + P_{JOIN} P_{JOIN}^{SUCC}\right) - P_{STAY} P_{STAY}^{SUCC}}$$

$$\pi_u = 1 - \frac{P_{START} + P_{JOIN} P_{JOIN}^{SUCC}}{1 + P_{STAY}\left(P_{START} + P_{JOIN} P_{JOIN}^{SUCC}\right) - P_{STAY} P_{STAY}^{SUCC}}$$

where π_e, π_s, π_u are the long run probabilities of being employed, self-employed and unemployed, and P_{JOIN}^{SUCC} is the solution to:

[18] $$P_{JOIN}^{SUCC} = \left(1 - \left(1 - \frac{p \cdot V^e}{A \cdot N^{JOIN}}\right)^A\right)$$

$$N_{JOIN} = N \cdot P_{JOIN}\left(1 - eP_{STAY}\right)$$

$$p = 1 - \left(1 - A / V^e\right)^{N_{JOIN}}$$

$$V^e = eN \cdot \frac{2\left(g_H - g_L\right)\left(1 - P_{STAY}\right) + \left(g_H^2 - g_L^2\left(1 - P_{STAY}\right)\right)}{2\left(g_H - g_L\right)} + N\left(1 - eP_{STAY}\right) P_{START} \bar{J}$$

$$e = 1 - u = \pi_E + \pi_S = \frac{P_{START} + P_{JOIN} P_{JOIN}^{SUCC}}{1 + P_{STAY}\left(P_{START} + P_{JOIN} P_{JOIN}^{SUCC}\right) - P_{STAY} P_{STAY}^{SUCC}}$$

The system is solved numerically. The figures below report the effects of the various parameters on individual choices, starting from a reference case with:

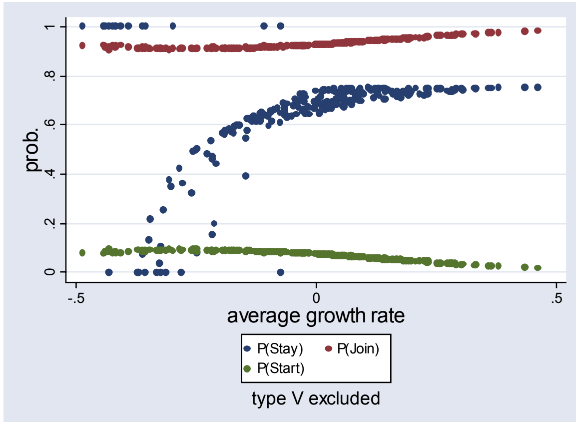
[19] $g_L = -.5 \qquad g_H = .5$
$s_L = -.5 \qquad s_H = .5$
$J_L = 0 \qquad J_H = 20$
$r = .75 \qquad \alpha = .01$
$A = 10 \qquad N = 1000$

Remember that $P_{STAY} \leq 1; P_{JOIN} + P_{START} + P_{RELAX} = 1$.

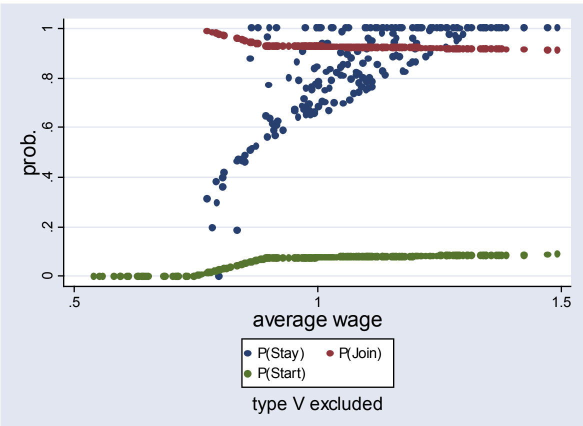
The effect of the reservation wage is linear (fig. 1a). Above a certain threshold, it starts lowering the probability of taking a Stay decision; then, as it approximates 1 it brings the probability of starting a new business to 0. An increasing average growth rate (fig. 1b) increases the probability of taking a Stay decision (as P_{STAY}^{SUCC} gets higher, i.e. there are more chances to be confirmed, once this decision is taken), and – once the worker has left – the probability to apply for a new job (as there are more vacancies; hence the probability of getting a new job P_{JOIN}^{SUCC} is higher). Higher average wages (fig. 1c) increase the chance of staying, but – above a certain threshold – do not influence the other probabilities. A greater value of the start-up sunk costs α (fig. 1d) has a positive effect on the probability of taking a Stay decision, and of course a negative effect on the probability of taking a Start decision. The number of applications that can be contemporarily sent out by workers does not affect significantly their choices (fig. 1e).

Figure 1: Choice probabilities

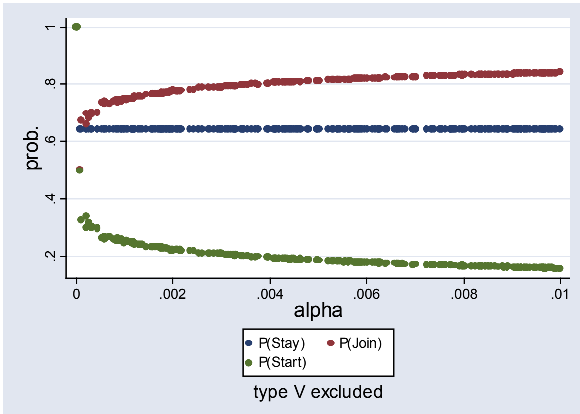
(a)



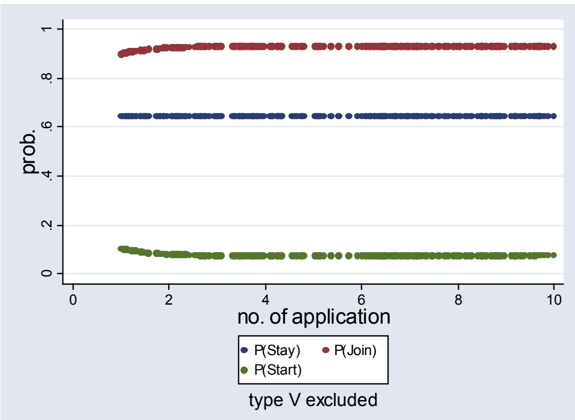
(b)



(c)



(d)



(e)

As for what regards the effects of the parameters on the final outcome, i.e. on the equilibrium state probabilities π_u, π_e, π_s and on the number of matches M, they are shown in figure 2, with reference to the benchmark case outlined above.

3.1 Reservation wage

Unemployment is affected by the reservation wage, when it is above a certain threshold, but the relationship may appear counterintuitive: a greater reservation wage lowers the unemployment rate. To explain this, note first that the values of the parameters do not allow for type V situations, i.e. the probability of staying out of the labour market (taking a Relax decision) is null. Hence, there remain only two ways of becoming unemployed: the first one is taking a Stay decision, but not being reconfirmed in the same job due to a negative conjuncture; the second one is taking a Join decision, but not being selected for any of the A applications sent out. However, the first risk does not depend on r, while the second one is decreasing in r, as the number of vacancies increases (the number of matches also increases, as depicted in fig. 2c). Since the probability of taking a Stay decision is decreasing in r, the resulting relationship between the reservation wage and the unemployment rate has to be negative. However, by allowing all parameters to change randomly (see eq. 20 below), it becomes evident that the probability of having a type V situation is increasing in r.

Table 8: Type occurrences

	type		
R	**2** (%)	**3** (%)	**5** (%)
0-.1	99.12	0.88	
.1-.2	98.44	1.56	
.2-.3	100.00		
.3-.4	100.00		
.4-.5	100.00		
.5-.6	99.24		0.76
.6-.7	94.12	0.74	5.15
.7-.8	86.07	2.46	11.48
.8-.9	73.87		26.13
.9-1	59.85		40.15

As the number of workers having voluntarily quitted their job increases with r, they first decide to start a new business on their own. Then, with r approaching 1, they find more convenient to start sending applications around. The resulting trend for the number of new start-ups is tent-shaped (fig. 2b).

3.2 Average growth rate

Higher expected growth rates increase the probability of being confirmed in the present job, thus increasing the probability of taking a Stay decision. This lowers the unemployment rate, the number of matches and the number of new start-ups at the same time (fig. 2d,e,f).

3.3 Average wage

A similar story holds for average wage (fig. 2g,h,i). Here, however, the correlation between the start-up rate and the average wage is somehow tent-shaped. High average wages increase the probability workers are satisfied with their present job, and thus reduce the incentive for starting their own business. However, low average wages increase the importance of the αJ sunk cost, and thus also reduce the likelihood of starting a new business.

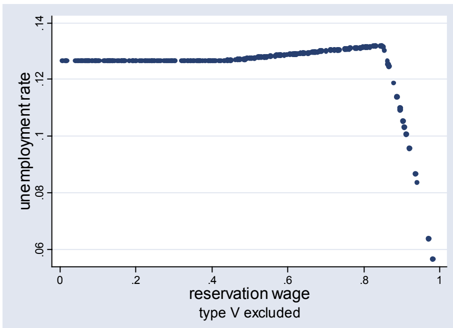
3.4 Start-up sunk costs

An increase in sunk costs lowers the incentives to start a new business, increasing the probability of applying for other jobs. Since the probability of taking a Stay decision remains unaffected, the total number of vacancies decreases. Hence the positive correlation with the unemployment rate (fig. 2j,k,l).

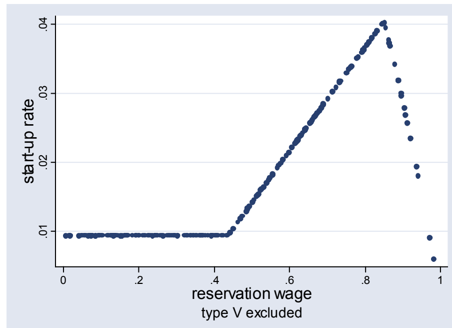
3.5 Number of contemporary applications

The effect of the number of applications A is the same as in Albrecht et al. (2003), with respect to the number of matches (fig. 2o). This model however also allows studying its effects on total unemployment and new businesses. A higher A increases the probability of taking a Join decision (by increasing the probability at least one application is selected), while lowering the probability of taking a Start decision. The overall effect on the total number of vacancies is decreasing in A, even if above a certain threshold this is slightly reversed. The unemployment rate follows this trend (since the number of people holding their jobs remains constant), while the figure for the number of vacancies looks reversed (fig. 2m,n).

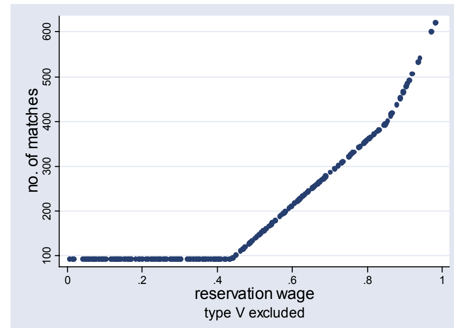
Figure 2: Effects of relevant parameters on the long-run equilibrium probabilities



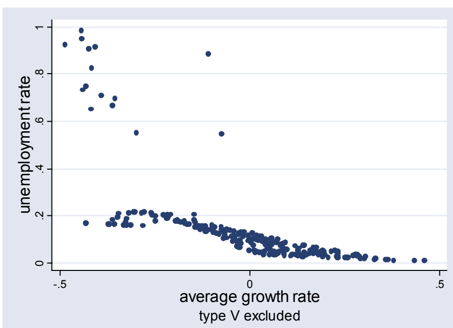
(a)



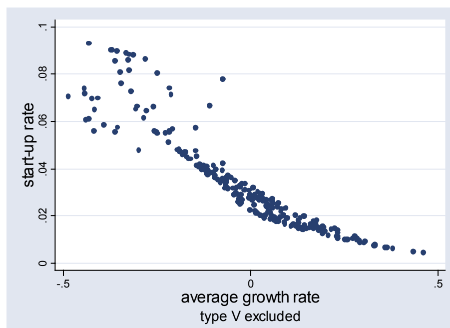
(b) % of non-stayers



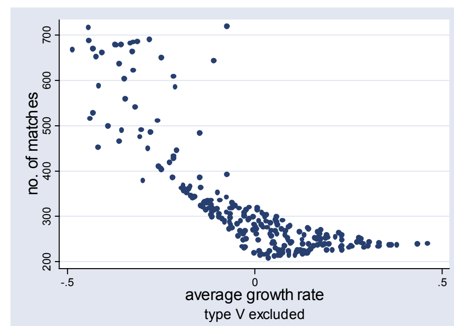
(c)



(d)

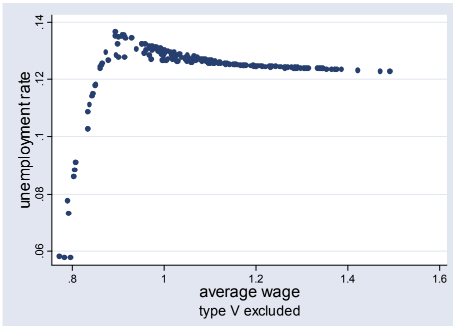


(e) % of non-stayers

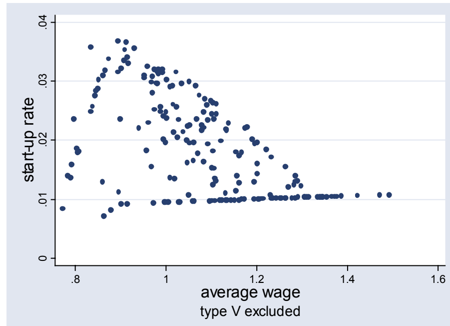


(f)

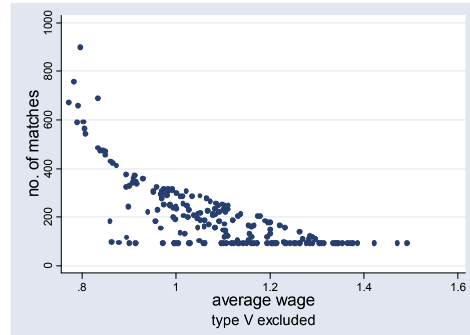
Figure 2 (cont'd): Effects of relevant parameters on the long-run equilibrium probabilities



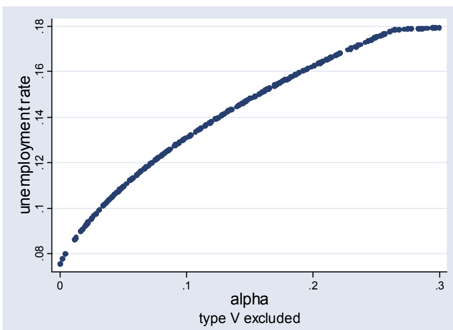
(g)



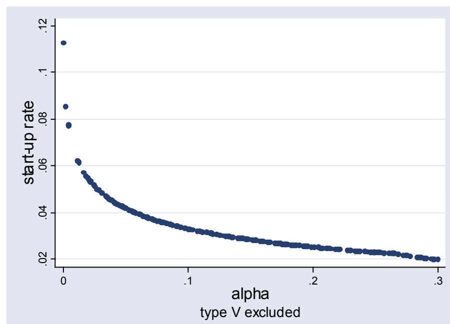
(h) % of non-stayers



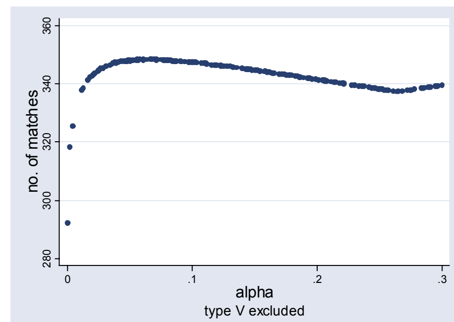
(i)



(j)

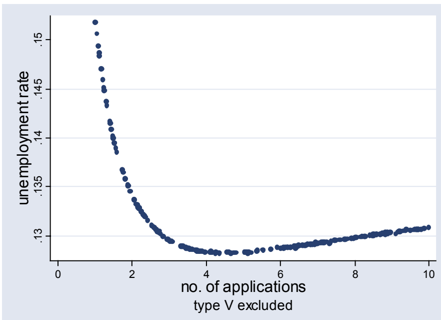


(k) % of non-stayers

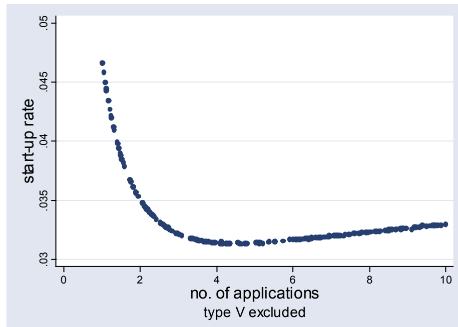


(l)

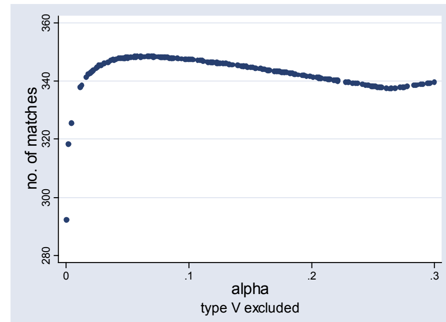
Figure 2 (cont'd): Effects of relevant parameters on the long-run equilibrium probabilities



(m)



(n) % of non-stayers



(o)

By allowing all parameters to vary, according to random extractions:

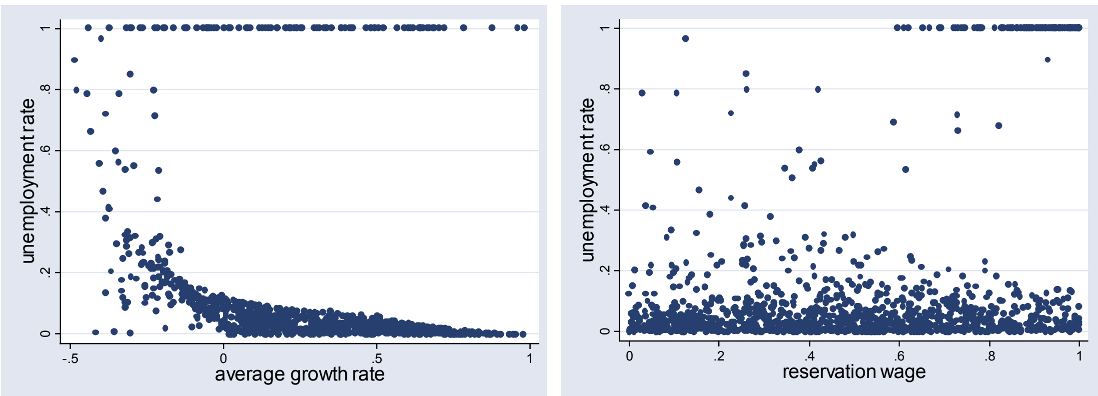
$$
\text{[20]} \quad
\begin{array}{ll}
g_L \sim \mathrm{U}(-.5,0) & g_H \sim \mathrm{U}(g_L, g_L + 1) \\
s_L \sim \mathrm{U}(-5,0) & s_H \sim \mathrm{U}(s_L, s_L + 1) \\
J_L = 0 & J_H \sim \mathrm{U}(0,50) \\
r \sim \mathrm{U}(0,1) & \alpha \sim \mathrm{U}(0,.3) \\
A = 10 & N = 1{,}000
\end{array}
$$

a further characterization of the long-run equilibrium is possible.

3.6 Unemployment rate

The unemployment rate remains clearly negatively correlated with the average growth rate (fig. 3a). The negative correlation with the reservation wage for type II and type III situations, described above, almost vanishes. Moreover, as the reservation wage becomes high enough, it starts increasing the probability of obtaining type V situations. Then, the expected positive correlation is found (fig. 3b).

Figure 3:Long-run unemployment rate



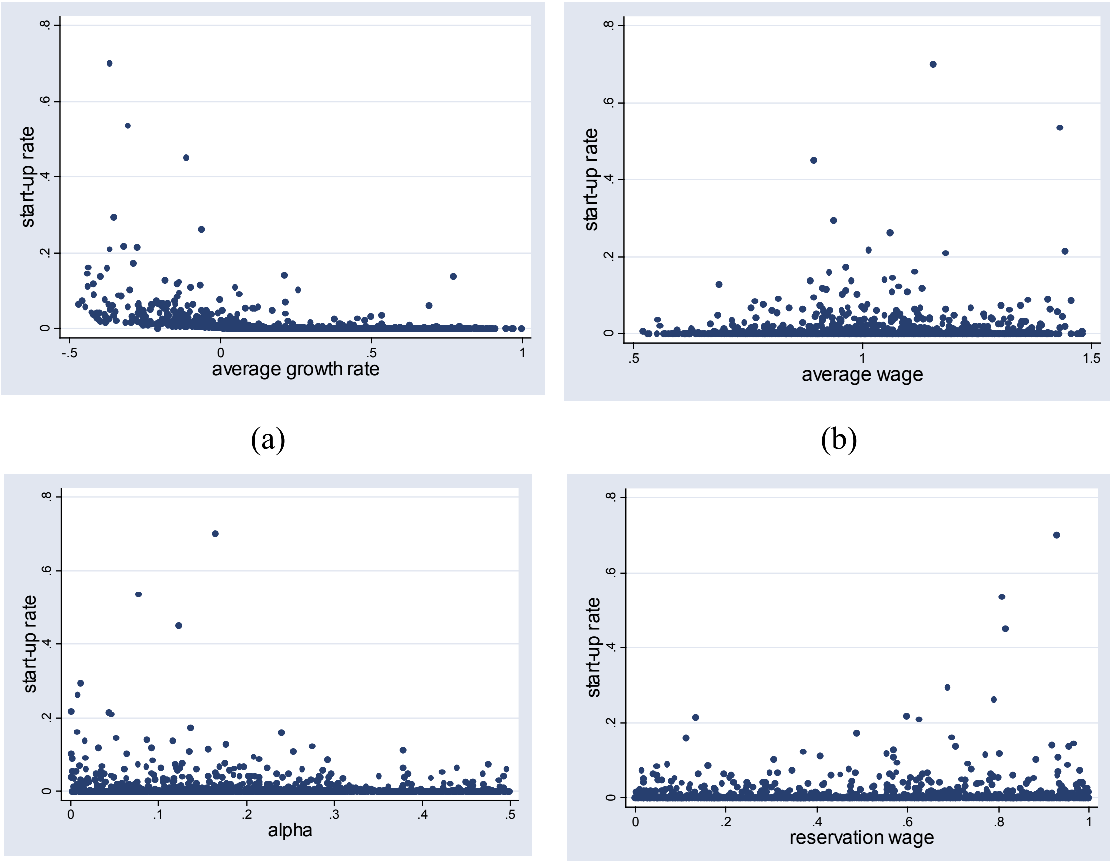
(a) (b)

3.7 New start-ups

When looking at the share of new businesses, the negative correlation with the average growth rate is again found. (fig. 4a), while the tent-shaped correlation with the average wage becomes a sort of bell-shaped figure (fig. 4b).

A greater α of course lowers the incentives for new start-ups (fig. 4c), while the effects of a greater reservation wage are no longer easily detectable (fig. 4d).

Figure 4: Long-run start-up rate



(a) (b)

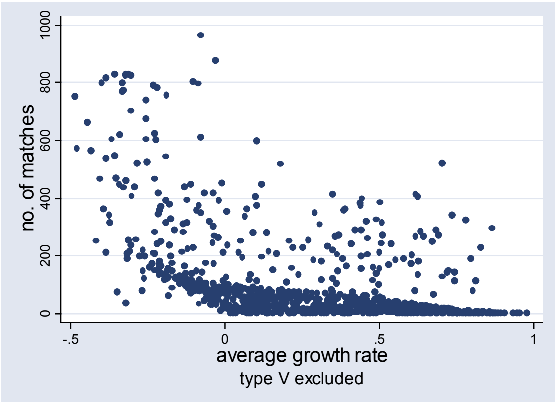
(c) (d)

% of non-stayers

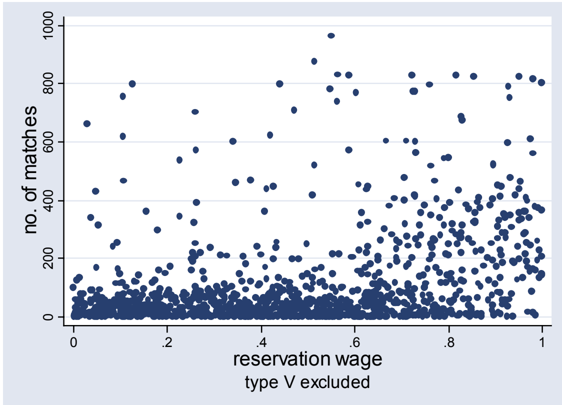
3.8 The matching function

Looking at the number of matches completes the picture. The correlation with the average growth rate is still clearly negative (workers are more likely to hold their job) (fig. 5a). The matching function still shows a positive dependence on the reservation wage, when it becomes big enough to influence individual choices (fig. 5b). However, it is no more possible to spot the negative correlation with the average wage, without controlling also for the other variables (fig. 5c).

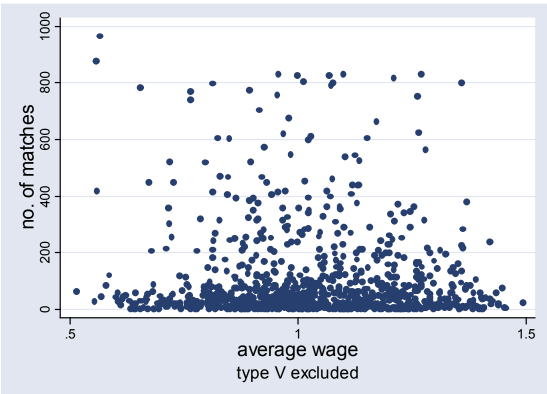
Figure 5: Long-run matches



(a)



(b)



(c)

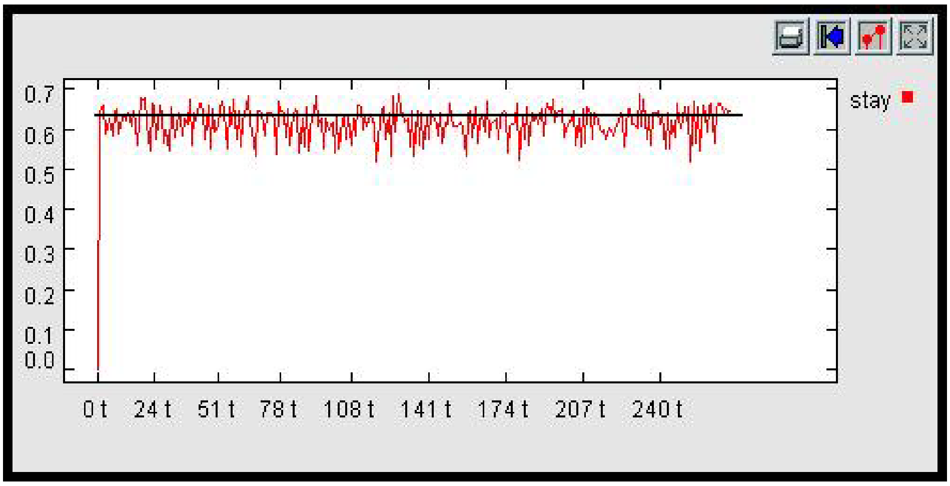
4. Firm demography

As opposed to standard search models, here vacancies are linked to firms, thus allowing for the analysis of firm demography. While new firms birth rate is given by the start-up probability derived above, firm size distribution and firm number (which is obviously given by the interaction of the birth and death rates) are explored by means of an agent-based simulation. Agent-based models are computer programs that simulate the behaviour of the basic entities in the system (i.e. workers, vacancies and firms), given some interaction rules. Aggregate behaviour is thus reconstructed 'from the bottom up'.

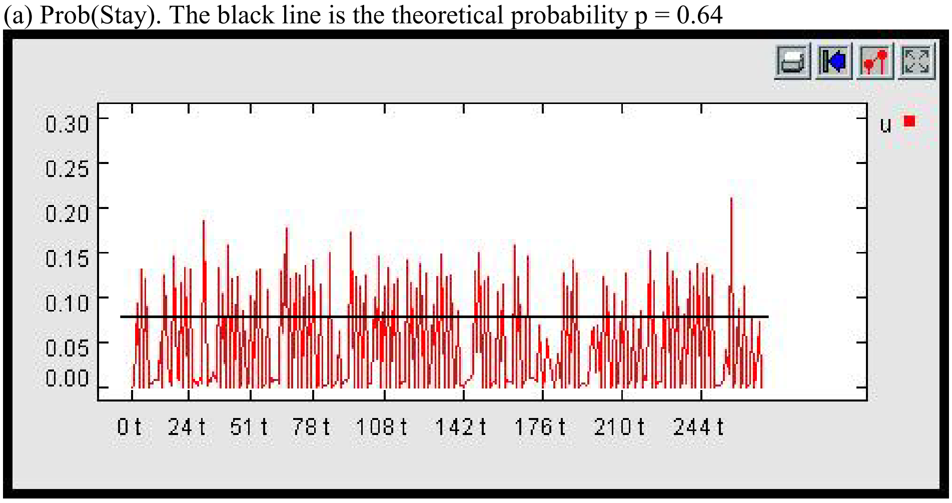
The choice of writing a simulation[3] has some implications, irrespective of the model specification. Generally, an analytical model is not immediately operational, i.e. the imaginary manual for playing the search game described by the model has to be worked out. This may induce some change in the model itself. In particular, due to the non-parallel discrete processing characteristics of most PC, the model must be sequential and cast in discrete time, as opposed to the analytical reference model. In addition to time, some other variables that are continuous in the analytical model (as the number of employees) have to be treated in units. Equilibrium relations cannot be used directly; rather, they have to be derived through non-equilibrium steps. For instance, the number of people expected to take a Join decision, which in the analytical model is the solution of an equilibrium equation involving rational expectations, is considered to be equal to the number of people taking a Join decision in the last period (adaptive expectations). Similarly, the expected number of vacancies is the number of vacancies observed in the last period. Therefore, the question whether this adaptive expectations version of the model converges towards any equilibrium at all, and whether this equilibrium is the same of the rational expectations version, naturally arises. However, it turns out that, but for some noise, the simulation model succeeds in recovering the equilibrium relations.

[3] The simulation is written in Java code, using JAS libraries (http://sourceforge.net/projects/jaslibrary/).

Figure 6: Analytical vs. simulation results (parameter values given in [19])



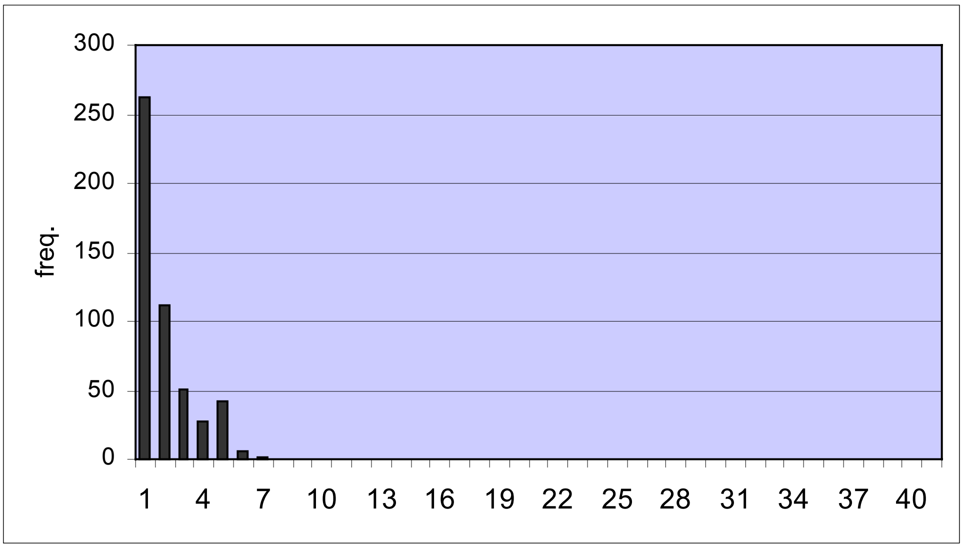
(a) Prob(Stay). The black line is the theoretical probability p = 0.64



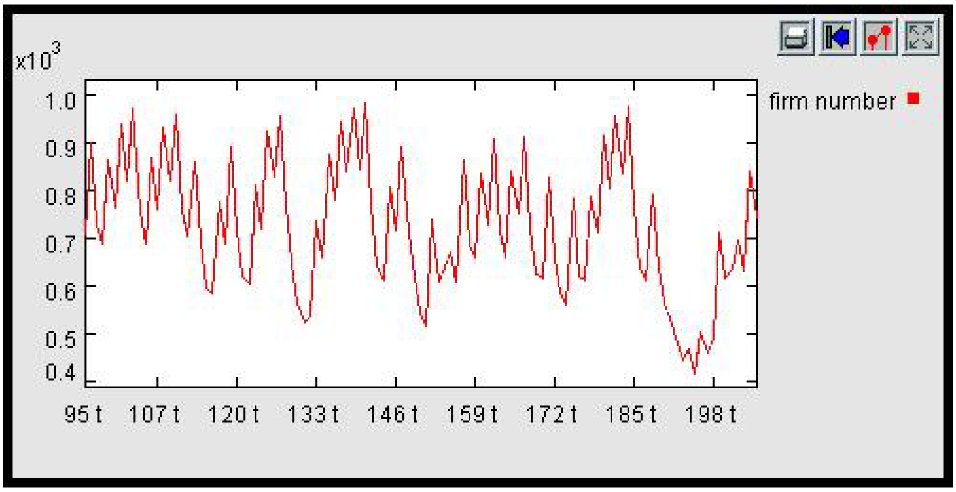
(b) Unemployment rate. The black line is the theoretical probability u = 0.08

It is then possible to use the simulation model to analyse firm demography. As an example, the resulting figures for the parameters values given in [19] are reported below:

Figure 7: Firm demography (parameter values given in [19])



(a) Firm size distribution at t=200



(b) Firm number

5. Extensions of the model

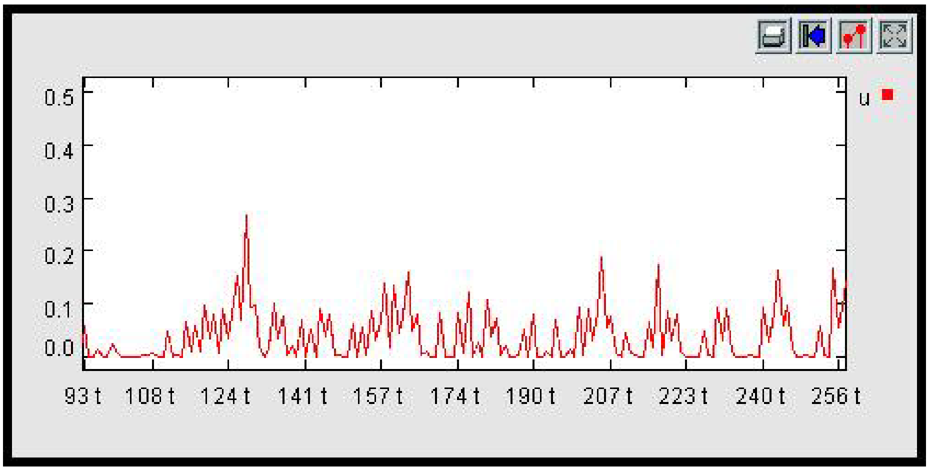
In this section, I deal with the relaxation of some assumptions of the model. In particular, some variations in the structure of the stochastic wage multiplier $s_{f,t}$ are considered. When s is correlated across firms or in time, or is dependent on the business cycle variable g, it becomes difficult to solve analytically for the probabilities of a successful Stay or Join decision, but for simple cases. For instance, when s is firm-specific i.e. $s_{f,t} = s_f$, workers always want to stay, once they are employed in a firm offering a high enough wage (which they will sooner or later find). They then become unemployed only if (randomly) fired, when the firm is experiencing negative growth. The fact that in more complicated cases the probability of a successful choice becomes difficult to compute has not merely analytical consequences. One could question whether real individuals could be thought of acting *as if* they were able to make such complex computations, in order to take the best choice. The realism of the model is thus challenged. When complex feedbacks are involved, it becomes more sensible to consider simpler individual choice rules, thus abandoning the realm of maximization in favour of a bounded rationality model of individual behaviour.

The first step is thus changing slightly the rules of the game:

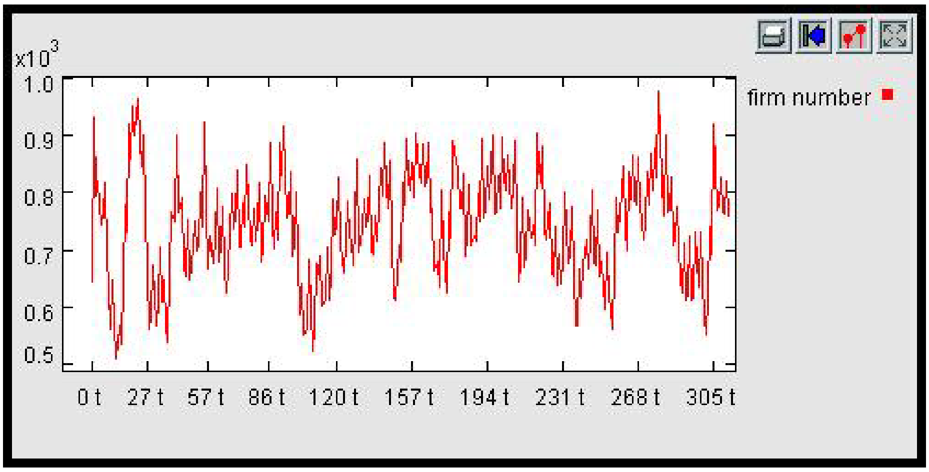
- Workers have adaptive expectations concerning their future wage, and they discount them for a simple proxy of the probability of being fired, the unemployment rate: $w_{STAY} = w_{f,t} \cdot e$.
- As for the expected payoff resulting from applying for other jobs, workers take the average wage of all employees, multiplied for the probability of one of their application being selected, which remains unchanged: $w_{JOIN} = \overline{w}_t \cdot P_{JOIN}^{SUCC}$. Note that the average wage of all employees may differ from $(1+\overline{s})$, since workers with a low s are more willing to change their job. The expected number of vacancies is again thought to be the same as in the last period.
- When considering the option to start a new business, workers expect a payoff equal to the average wage of all entrepreneurs, net of the start-up costs: $w_{START} = \overline{w}_{START,t} - \alpha J$

These rules are simple variations of those of the analytical model, which trade off optimality for computability and simplicity. When combined with firm- and time-specific wage shocks $s_{f,t}$ they typically produce cycles. These cycles are characterized both by periods of sharp decline in the number of active firms, and consequent steep rise of the unemployment rate, and by periods in which the number of firms and the unemployment rate ‘breath’ in and out more regularly.

Figure 8: Outcome of non-optimising model (parameter values given in [19])



(a) Unemployment rate



(b) Firm number

Overall, these results do not differ much from those of the optimising model, although the dynamics appear a little more well-behaved. The analytical model is thus shown to be robust to its operationalization, and to small departures from optimising behaviour.

Having a robust model of individual behaviour, it is now possible to add some structure to the stochastic wage multiplier $s_{f,t}$. Among the many possible variations, two simple extensions of the benchmark model are presented here.

5.1 Auto-correlation of start-up profits

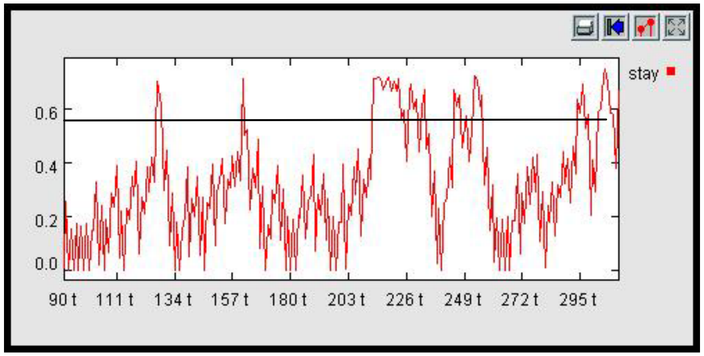
Suppose start-ups do not get their $s_{f,t}$ from the D_s distribution, but rather from the actual distribution of other start-uppers. This may cause a self-sustaining process: following some particularly high extraction of the $s_{f,t}$ among the first start-uppers, expectations of start-up profits will rise, hence producing more start-ups, which will also enjoy high profits. However, this will slowly raise the average s, thus leading, in conjunction with a decreased unemployment rate, to a higher probability of Stay decisions. Eventually, the number of start-uppers will decrease, thus making it easier for unlucky low-profit start-ups to impact the average start-up profits. A new period characterized by few low-wage start-ups can start, and last until a new generation of lucky new businesses will appear.

Typical results for this model are reported in the figure below. Parameter values are:

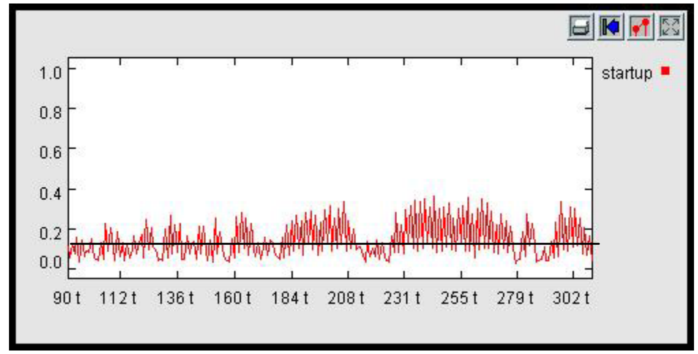
[21]
$$\begin{array}{ll} g_L = -.5 & g_H = .5 \\ s_L = -.5 & s_H = .5 \\ J_L = 0 & J_H = 10 \\ r = .80 & \alpha = .1 \\ A = 10 & N = 1000 \end{array}$$

The black line on each graph represents the equilibrium value in the analytical model. The dynamics look now more complex, with periods of high unemployment alternating to periods with almost full employment. Moreover, many combinations of the parameter values give rise to either full employment or full unemployment situations, which become very stable, once established.

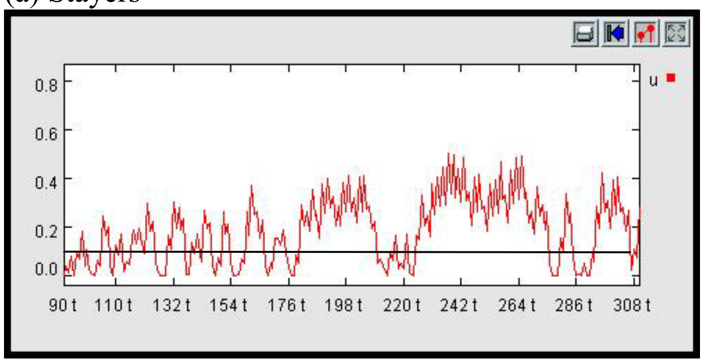
Figure 9: Outcome of model 5.1 (parameter values given in [21])



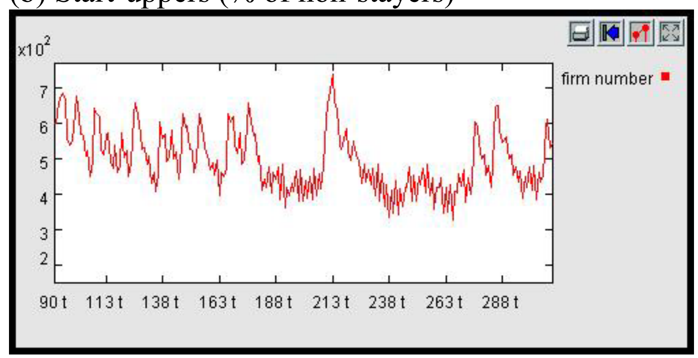
(a) Stayers



(b) Start-uppers (% of non-stayers)



(c) Unemployment rate



(d) Number of firms

Black lines represents equilibrium values in the reference (analytical) model

5.2 Correlation of profits and business cycle shocks

This last model enriches the previous one, by adding autocorrelation in the business cycle and letting wages to depend on the business cycle. More precisely, each period the new business cycle random shock g_t is averaged with its past value, i.e.:

$$[22] \quad g_{t+1} = \frac{g_t + U(g_L, g_H)}{2}$$

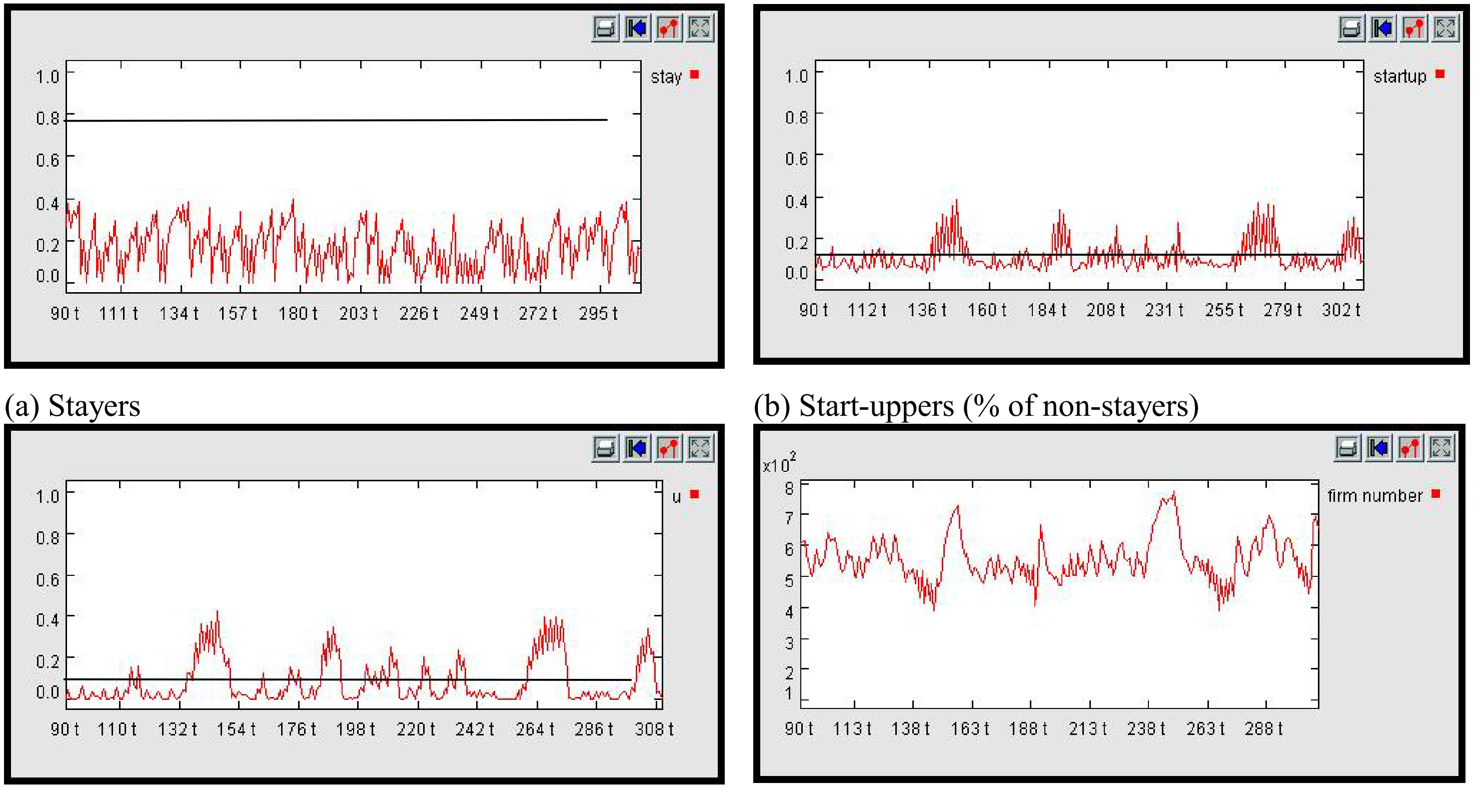
The start-up expected and actual profits are computed as in the model of section 5.1, while for existing firms the wage shock is interacted with the business cycle in the following way:

$$[22] \quad s_{f,t+1} = U(s_L, s_H) \cdot (1 + g_{t+1})$$

As before, the actual extraction of $U(s_L,s_H)$ is known to employees at time t, but the extraction g_{t+1} is not.

A typical outcome is reported in figure 10. The same values of the parameters as in the previous section were used. Again, subsets of the parameters space lead to very stable and polarized (either full employment or full unemployment) equilibria.

Figure 10: Outcome of model 5.2 (parameter values given in [21])



(a) Stayers

(b) Start-uppers (% of non-stayers)

(c) Unemployment rate

(d) Number of firms

Black lines represents equilibrium values in the reference (analytical) model

6. Conclusions

In this paper I have provided an analytical model of (two-sided) search in the labour market, with optimising individuals. With respect to the previous literature on the topic, this model allows the joint investigation of unemployment and firm dynamics, by explicitly considering the vacancy generation process of firms. The convergence of the model to the equilibrium is tested through an agent-based simulation, which also shows that a non-optimising but more realistic version of the model leads to basically the same results. This bounded rationality version of the model is then used to investigate the effects of different (and more realistic) assumptions about wages and the business cycle.

Overall, the simulation models show that, by adding realistic features to the *behaviour* of the individuals and to the *structure* of the model, it is relatively easy to obtain more interesting dynamics, as compared to those of the reference model. While small changes towards more realistic models of individual behaviour do not significantly alter the outcome, thus showing the robustness of the benchmark model, small changes in its structure may lead to outcomes that bear little resemblance with those of origin. More detailed investigations of the latter models, and of how they are related with the basic one, are left for future research.

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Appendix – Choice probabilities

$$\boxed{\begin{aligned} P_{JOIN} &= \Pr(w^e_{JOIN} \geq w^e_{START}, w^e_{JOIN} \geq r) = \\ &= \Pr\left(w^e_{JOIN} \geq w^e_{START}\right) \cdot \Pr\left(w^e_{JOIN} \geq r \mid w^e_{JOIN} \geq w^e_{START}\right) \end{aligned}}$$

$$\Pr\left(w^e_{JOIN} \geq w^e_{START}\right) = \Pr\left(J \geq \frac{(1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)}{\alpha} = k\right) =$$

$$= \begin{cases} 0 & \text{if} \quad k \geq J_H \Leftrightarrow P^{SUCC}_{JOIN} \leq \dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}} \\ \dfrac{\alpha J_H - (1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)}{\alpha\left(J_H - J_L\right)} & \text{if} \quad k \in \left[J_L, J_H\right] \Leftrightarrow P^{SUCC}_{JOIN} \in \left[\dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}, \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}\right] \\ 1 & \text{if} \quad k \leq J_L \Leftrightarrow P^{SUCC}_{JOIN} \geq \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}} \end{cases}$$

$$\Pr\left(w^e_{JOIN} \geq r \mid w^e_{JOIN} \geq w^e_{START}\right) = \Pr\left((1+\bar{s})P^{SUCC}_{JOIN} \geq r \,\middle|\, J \geq \frac{(1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)}{\alpha}\right) =$$

$$= \begin{cases} 0 & \text{if} \quad P^{SUCC}_{JOIN} < \dfrac{r}{1+\bar{s}} \\ 1 & \text{if} \quad P^{SUCC}_{JOIN} \geq \dfrac{r}{1+\bar{s}} \end{cases}$$

$$\boxed{\begin{aligned} P_{START} &= \Pr(w^e_{START} > w^e_{JOIN}, w^e_{START} \geq r) = \\ &= \Pr\left(w^e_{START} > w^e_{JOIN}\right) \cdot \Pr\left(w^e_{START} \geq r \mid w^e_{START} > w^e_{JOIN}\right) \end{aligned}}$$

$$\Pr\left(w^e_{START} > w^e_{JOIN}\right) = \Pr\left(J < \frac{(1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)}{\alpha} = k\right) =$$

$$= \begin{cases} 0 & \text{if} \quad k \leq J_L \Leftrightarrow P^{SUCC}_{JOIN} \geq \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}} \\ \dfrac{(1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)-\alpha J_L}{\alpha\left(J_H - J_L\right)} & \text{if} \quad k \in \left[J_L, J_H\right] \Leftrightarrow P^{SUCC}_{JOIN} \in \left[\dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}, \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}\right] \\ 1 & \text{if} \quad k \geq J_H \Leftrightarrow P^{SUCC}_{JOIN} < \dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}} \end{cases}$$

$$\Pr\left(w^e_{START} \geq r \middle| w^e_{START} > w^e_{JOIN}\right) = \Pr\left(J \leq \frac{1+\bar{s}-r}{\alpha} \middle| J < \frac{(1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)}{\alpha}\right) =$$

$$= \begin{cases} 0 & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} \leq J_L \\ \dfrac{(1+\bar{s}-r)-\alpha J_L}{(1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)-\alpha J_L} & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} \in \left[J_L, J_H\right] \\ & \quad \& \quad k \leq J_H \Leftrightarrow P^{SUCC}_{JOIN} \geq \dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}} \\ 1 & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} \in \left[J_L, J_H\right] \\ & \quad \& \quad k \geq J_H \Leftrightarrow P^{SUCC}_{JOIN} < \dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}} \\ 1 & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} \geq J_H \end{cases}$$

$$\boxed{\begin{aligned} P_{RELAX} &= \Pr(\max(w^e_{JOIN}, w^e_{START}) < r) = \\ &= \Pr\left(r > w^e_{JOIN}\right) \cdot \Pr\left(w^e_{JOIN} \geq w^e_{START} \mid r > w^e_{JOIN}\right) + \\ &+ \Pr\left(r > w^e_{START}\right) \cdot \Pr\left(w^e_{START} \geq w^e_{JOIN} \mid r > w^e_{START}\right) \end{aligned}}$$

$$\Pr\left(r > w^e_{JOIN}\right) = \Pr\left(r > (1+\bar{s})P^{SUCC}_{JOIN}\right) =$$

$$= \begin{cases} 0 & \text{if} \quad P^{SUCC}_{JOIN} \geq \dfrac{r}{1+\bar{s}} \\ 1 & \text{if} \quad P^{SUCC}_{JOIN} < \dfrac{r}{1+\bar{s}} \end{cases}$$

$$\Pr\left(w^e_{JOIN} \geq w^e_{START} \middle| r > w^e_{JOIN}\right) = \Pr\left(J \geq \frac{(1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)}{\alpha} = k \middle| P^{SUCC}_{JOIN} < \frac{r}{1+\bar{s}} \right) =$$

$$= \begin{cases} 0 & \text{if} \quad k \geq J_H \Leftrightarrow P^{SUCC}_{JOIN} \leq \dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}} \\ \dfrac{\alpha J_H - (1+\bar{s})\left(1-P^{SUCC}_{JOIN}\right)}{\alpha(J_H - J_L)} & \text{if} \quad k \in [J_L, J_H] \Leftrightarrow P^{SUCC}_{JOIN} \in \left[\dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}, \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}\right] \\ 1 & \text{if} \quad k \leq J_L \Leftrightarrow P^{SUCC}_{JOIN} \geq \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}} \end{cases}$$

$$\Pr\left(r > w^e_{START}\right) = \Pr\left(J > \frac{1+\bar{s}-r}{\alpha} \right) =$$

$$= \begin{cases} 0 & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} \geq J_H \\ \dfrac{\alpha J_H - (1+\bar{s}-r)}{\alpha(J_H - J_L)} & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} \in [J_L, J_H] \\ 1 & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} < J_L \end{cases}$$

$$\Pr\left(w^e_{START} \geq w^e_{JOIN} \middle| r > w^e_{START}\right) = \Pr\left(J \geq \frac{(1+\bar{s})(1-P^{SUCC}_{JOIN})}{\alpha} = k \middle| P^{SUCC}_{JOIN} < \frac{r}{1+\bar{s}}\right) =$$

$$= \begin{cases} 0 & \text{if} \quad k \leq J_L \Leftrightarrow P^{SUCC}_{JOIN} \geq \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}} \qquad \text{or :} \\ & \quad \dfrac{1+\bar{s}-r}{\alpha} \in [J_L, J_H] \quad \text{and} \quad k \leq \dfrac{1+\bar{s}-r}{\alpha}, \text{ i.e.} \\ & \quad \dfrac{1+\bar{s}-r}{\alpha} \in [J_L, J_H] \quad \text{and} \quad P^{SUCC}_{JOIN} \geq \dfrac{r}{1+\bar{s}} \\ \dfrac{(1+\bar{s})(1-P^{SUCC}_{JOIN}) - \alpha J_L}{\alpha(J_H - J_L)} & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} \leq J_L \quad \text{and} \quad k \in [J_L, J_H], \text{ i.e.} \\ & \quad \dfrac{1+\bar{s}-r}{\alpha} \leq J_L \quad \text{and} \quad P^{SUCC}_{JOIN} \in \left[\dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}, \dfrac{1+\bar{s}-\alpha J_L}{1+\bar{s}}\right] \\ \dfrac{r-(1+\bar{s})P^{SUCC}_{JOIN}}{\alpha J_H - (1+\bar{s}-r)} & \text{if} \quad \dfrac{1+\bar{s}-r}{\alpha} \in [J_L, J_H] \quad \text{and} \quad k \in \left[\dfrac{1+\bar{s}-r}{\alpha}, J_H\right], \text{ i.e.} \\ & \quad \dfrac{1+\bar{s}-r}{\alpha} \in [J_L, J_H] \quad \text{and} \quad P^{SUCC}_{JOIN} \in \left[\dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}}, \dfrac{r}{1+\bar{s}}\right] \\ 1 & \text{if} \quad k \geq J_H \Leftrightarrow P^{SUCC}_{JOIN} \leq \dfrac{1+\bar{s}-\alpha J_H}{1+\bar{s}} \end{cases}$$