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## **Active and Passive Policies Against Poverty with Decreasing Employability**

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## Abstract

In this paper we propose a non-equilibrium model in order to explain the search behavior of unemployed workers. This modeling strategy, framed in a rational choice paradigm, allows us to investigate the effects of negative duration dependence in the out-of-unemployment hazard rate, accounting for a decrease in employability as the unemployment spell lengthens. We show that individuals react to an expected reduction in their search effectiveness by increasing their search efforts. We then analyze active and passive labor market policies, consisting in training programs and income support schemes. We show that it is optimal for the government to include among the eligibility criteria for subsidized training a minimum length of the unemployment spell. However, it is optimal to recruit workers before they become discouraged and stop searching. We also show that for a broad range of the parameters the optimal income support scheme takes the form of unemployment benefits granted for a limited amount of time starting from the beginning of the unemployment spell, coupled with social assistance for long-term unemployed with very limited residual employability.

## 1 Introduction

It is well known that income support schemes to fight poverty might actually dynamically exacerbate the problem, as they also have an effect on the labor supply decision of individuals and on their ability to find a job.

In their influential work on welfare dependence, Bane and Ellwood (1994) provide a theoretical framework for the behavioral models underlying the potential detrimental role of welfare. They discuss three models: the *rational choice*

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model, the *expectancy* model and the *cultural* model. The rational choice paradigm motivates most of the empirical studies on work disincentives, i.e. the negative effects of welfare transfers on labor supply. Individuals subject to the constraints given by actual work opportunities and welfare policies, choose whether and how much to work following the principle of maximization of utility: if the benefit is rather convenient with respect to work, individuals prefer welfare and stay out of the labor market. Long-term welfare participation follows as a result of a series of conscious and rational choices. The latter models relate instead to individuals' self-esteem and to the interaction of agents behavior (living in environments where most of the people rely on welfare instead of work may favor the change of attitudes and mores, and the development of a different system of values).

Aside of the potentially detrimental effect of the amount of benefits, Bane and Ellwood (among others) recognize that the duration of economic subsidies paid out by public assistance is also crucial. It is generally thought that systems providing long term benefits are costly and not efficient. Moreover, according to the "welfare trap" theory, long duration of interventions may itself constitute a cause of poverty, as recipients may develop dependence on the subsidy, making it more difficult to become autonomous from public support.

Most of the empirical studies (Dazinger, Haveman and Plotnick, 1981; Ermish and Wright, 1990; Hoynes, 1996; see also Moffit, 1992, Moffit, 2002 and Blank, 2002 for extensive surveys) – largely referred to the U.S. program AFDC<sup>1</sup> – exploit time and cross-state variation in the benefit level<sup>2</sup> for identification of the impact of the programs. This body of work confirms that transfer programs considerably reduce work effort. More recently, research has dealt with the impact of reforms of social assistance on labor supply: work requirements, sanctions, time limits, education or training programs programs (for example: Moffit, 2003; Van den Berg, Van der Klaauw and Van Ours; 2004).

Another strand of the literature concentrates on the effect of the level of benefits (or other specific features of the program) on welfare participation and on entry/exit rates from welfare. As regards exit rates, the aim is to assess the indirect effect of the benefit level on welfare spell length - if people reduce work effort because of the subsidy, time on welfare should increase (Blank, 1989; Hoynes e MaCurdy, 1994, Fortin, Lacroix and Drolet, 2004). Overall, there is a robust empirical evidence that a rise in the amount of the subsidy increases welfare participation and entry rates, while exit rates drop. Other studies focus on the assessment of negative duration dependence (some examples are Blank, 1989; Dahl, Lorentzen, 2003; Chay et al, 2005).

Active labor market policies have been developed to avoid the detrimental effects of simple income support schemes. The idea is to "bring people back

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<sup>1</sup>*Aid to Families with Dependent Children*, the main income support for the poor in the USA, which was specifically designed to support lone mothers with children. The program was replaced in 1996 by the more restrictive program *Temporary Aid for Needy Families*.

<sup>2</sup>the differences among states in the potential maximum benefit level under the AFDC program were very marked; moreover this level underwent significant changes in the 1960s to '80s period (Moffit, 1992)

to work” – whenever possible – as the best, and most cost-effective, way to fight poverty (Heckman et al., 1999). Training programs differ along a number of dimensions. They may consist in remedial basic education – for instance in reading and math – in vocational training or in short-term subsidized work experiences at government or non-profit agencies. Other programs (which fall outside the interest of this paper) may be directed toward employed individuals as on-the-job training, or involve job search assistance, including resume and interview preparation and help in focusing search strategy (see Barnow, 1989, for a comprehensive list). Empirical investigations have focused both on income gains (Friedlander et al., 1997) and on increases in employability (Orr et al., 1996). With respect to income effects the evidence suggests modest but positive effects but also modest costs, hence a significant social rate of return. This is true especially for particular groups of people (like adult women), while the impact for other groups (like adult men or youth) seems to be highly uncertain. In terms of increasing job finding rates for long-term unemployed, there is ample evidence that active labor market programs have small and in some cases even adverse effects. Some programs do provide positive incentives, although quite unintended. When mandatory, the threat of program participation can be so harsh that some individuals increase their search activity and/or lower their reservation wages in order to find a job before the program starts. When voluntary, the issue of the determinants of program participation arises (Heckman and Smith, 2004).

The rational choice approach to the analysis of the interactions between unemployment, welfare and poverty rests on the equilibrium analysis of the labor market provided by search models (see Rogerson et al, 2005 for a survey). In particular, Pissarides (2000, ch. 5) provides the standard equilibrium search model with varying search intensity. In that model, a worker continually chooses his search intensity  $s$  with increasing and strictly convex disutility  $c(s)$ . Job offers arrive according to a continuous time Poisson process with arrival rate  $s\eta(\theta)$ , where  $\theta$  measures labor market tightness and  $\eta$  is an increasing function. Search intensity can thus be thought of simply as “technical change” in the job matching technology summarized by the matching function. Workers choose their search effort in order to maximize their utility; firms choose how many vacancies to open in order to maximize their profits; the free entry, zero profit condition is imposed. The result is that an increase of the utility in unemployment (for instance due to unemployment benefits), a decrease of the expected utility in employment (such as a decrease in the expected wage) and an increase in the cost of search unambiguously lower search intensity, thus increasing the equilibrium unemployment rate and the unemployment duration.

We start from a model similar in spirit, but we develop it along completely different lines. First of all, we are not interested in general equilibrium analysis. We focus on a restricted group of workers, namely those more at risk of poverty. Hence, we can assume their work supply decision has a negligible impact on any individual probability of finding a job. Moreover, the decision on the part of the firms on how many vacancies to open is largely independent, and we consider it as exogenous. We are thus left with a much simpler, partial equilibrium

model, that we can then complicate by considering that search effectiveness might change in a foreseeable way over time. For instance, individuals might expect a macroeconomic downturn, or they might anticipate that their ability to find a job will deteriorate as the unemployment spell becomes longer. The latter might be due to a loss of skills, to a change in the structure and dimension of the network of people the unemployed person can contact, to stigma effects, etc. Note that standard equilibrium search models *à la* Pissarides can only account for unexpected changes in the environment. As individuals realize that the economy has entered a downturn, they immediately react by lowering their search effort. This is quite unsatisfactory since it yields

the counterfactual prediction that labor market participation and other measures of search intensity should be strongly procyclical and unemployment should be acyclical or even procyclical (Shimer, 2003).

Indeed, Shimer – along the lines of Stigler (1961) – has developed a discrete time equilibrium search model where search intensity is modeled as a worker’s choice of the number of simultaneous applications to make. He gets the main result that when the cost of making an application is small, workers with high employability react to an adverse shock by increasing their search intensity, while workers with low employability become discouraged, reducing their search intensity. In this model however downturns affect the productivity of filled jobs and the (exogenous) separation rate, and not the search effectiveness (which is endogenously determined). Here on the contrary we treat the search effectiveness as exogenous: a decrease in this parameter can thus be attributed both to a decrease in the employability of the individual or to a worsening of the general conditions of the economy. We will show that unemployed workers react to an expected decrease in their search effectiveness by increasing their search effort to exploit the most out of good times.

From a modeling perspective, letting search effectiveness vary over time in a systematic way impedes from imposing stationarity conditions, and forces us to a non-equilibrium analysis.

We then turn to investigating two policies aimed at contrasting poverty. One is an (active) program of training, that allows individuals to upgrade their skills and hence improve their subsequent employability. The other is a (passive) income support scheme, that can take the form of either unemployment or poverty benefits. Unemployment benefits are deemed to end after a few periods in unemployment; poverty benefits might become permanent.

As for the first measure, we characterize the individual and social optimal timing for undertaking a training program: we show that they do not generally coincide and hence that it might be optimal for an unemployed worker to turn down an offer to undertake a training course, even if it is for free (provided that the unemployed expects the offer to be repeated in the future). This might help explaining the low attendance rate of public training courses, as experienced in many European countries.

As for what concerns benefits, we show that it is optimal for the government to offer short-lived unemployment benefits, then leave the unemployed worker on his/her own for a while, and finally let the public assistance to take place when employability falls below a minimum threshold.

The rest of the paper is structured as follows. Section 2 describes the basic model with varying search effectiveness, and shows that when an unemployed worker expects his/her search effectiveness  $\gamma$  is going to decrease at time  $t + 1$ , the likelihood of an active search increases. Section 3 adds structure by considering a linear decay in search effectiveness. Section 4 is devoted to the analysis of training courses that allow the unemployed to re-gain employability, while section 5 offers a unified approach to the analysis of passive welfare policies like unemployment benefits and minimum income schemes. Section 6 summarizes and concludes.

## 2 The model

The value  $V_U$  of being unemployed at time  $t$  is given by:

$$V_{U,t} = U_t(s_t) + R[(1 - p_t(s_t))V_{U,t+1} + p_t(s_t)V_{E,t+1}] \quad (1)$$

where  $U_t$  is the utility in unemployment and depends on the search effort  $s_t$ ,  $p_t = \gamma_t s_t$ ,  $\gamma \in [0, 1]$  is the probability of getting a job in the period as a function of both search effectiveness  $\gamma_t$  and search intensity  $s_t$  and  $V_{E,t+1}$  is the value of being employed once found a job.  $R$  is the intertemporal discount factor. Analogously, the value  $V_E$  of being employed at time  $t$  is given by:

$$V_{E,t} = E_t + R[(1 - \delta)V_{E,t+1} + \delta V_{U,t+1}] \quad (2)$$

where  $E_t$  is the utility in employment (as a function of the wage  $w$  and other characteristics of the job) and  $\delta$  is the exogenous separation probability. We assume that there is no on-the-job search.

Such a model would be generally solved in equilibrium, by imposing the stationarity conditions  $V_{U,t} = V_U$  and  $V_{E,t} = V_E$ . Then, the value of  $V_U$  would be maximized with respect to  $s$  (possibly imposing an internal solution), thus identifying the optimal search strategy given search effectiveness  $\gamma$ .

Since search effectiveness generally depends on (i) how many vacancies are open, and (ii) how many unemployed workers compete for these vacancies, in a general equilibrium search models we would also look at what happens on the firms' side. By imposing some equilibrium condition such as zero profits, the optimal number of vacancies would be derived. Search effectiveness would then follow, allowing the model to close.

Here on the contrary we assume that  $\gamma_t$  is not constant over time. We do not assume any particular evolution for  $\gamma_t$ : however, we suppose that a (sufficiently long) employment spell acts as a *reset event*, bringing  $\gamma$  to some original value. If we measure time as time in unemployment (i.e. beginning at the start of the unemployment spell), this value is labeled  $\gamma_0$ . Note that  $V_{E,t}$  in eq. 2

depends on the value of future unemployment spells, but crucially not on the current search intensity  $s_t$ . If employment acts as a reset event, we can then simply write  $V_E = E + R[(1 - \delta)V_E + \delta V_0]$ , independent of time. Substituting recursively in eq. 1 and stopping, for clarity, at time  $t = 3$ , we obtain:

$$\begin{aligned}
V_{U,t} &= U_t + U_{t+1}R(1 - p_t) + U_{t+2}R^2(1 - p_t)(1 - p_{t+1}) \\
&+ U_{t+3}R^3(1 - p_t)(1 - p_{t+1})(1 - p_{t+2}) \\
&+ V_ERp_t + V_ER^2(1 - p_t)p_{t+1} + V_ER^3(1 - p_t)(1 - p_{t+1})p_{t+2} \\
&+ V_ER^4(1 - p_t)(1 - p_{t+1})(1 - p_{t+2})p_{t+3} \\
&+ R^4(1 - p_t)(1 - p_{t+1})(1 - p_{t+2})(1 - p_{t+3})V_{U,t+4} \\
&= U_t + U_{t+1}R(1 - p_t) + U_{t+2}R^2(1 - p_t)(1 - p_{t+1}) \\
&+ U_{t+3}R^3(1 - p_t)(1 - p_{t+1})(1 - p_{t+2}) \\
&+ V_ERp_t + V_ER^2(1 - p_t)p_{t+1} + V_ER^3(1 - p_t)(1 - p_{t+1})p_{t+2} \\
&+ V_ER^4(1 - p_t)(1 - p_{t+1})(1 - p_{t+2})p_{t+3} \\
&+ \epsilon
\end{aligned} \tag{3}$$

with  $\epsilon$  going to 0 as we proceed with expliciting  $V_{U,t}$ . This expression might be generalized in:

$$V_{U,t} = U_t + \sum_{i=1}^{\infty} R^i U_{t+i} \prod_{j=0}^{i-1} (1 - p_{t+j}) + V_E \sum_{i=1}^{\infty} R^i p_{t+i-1} \prod_{j=0}^{i-1} (1 - p_{t+j-1}) \tag{4}$$

with  $p_{t-1} = 0$ . This general expression won't be used any longer, as focusing instead on eq. 3 is more intuitive. As  $p_t$  increases due to an increase in the search intensity  $s_t$  the only term that increases is  $V_ERp_t$ , while everything else diminishes<sup>3</sup>. If we assume that  $U$  is linear in  $s$  the optimal level of  $s_t$  is thus a corner solution, either 0 or 1. The switch between these two cases takes place when  $V_{U,t}(s = 0) = V_{U,t}(s = 1)$ . Hence, substituting in eq. 1:

$$\gamma_t^* = \frac{U(0) - U(1)}{R(V_E - V_{U,t+1})} \tag{5}$$

where  $U(0) = U(s = 0)$  and  $U(1) = U(s = 1)$ .

The unemployed worker undertakes active search when  $\gamma_t$  is above the threshold and rests idle when it is below.

## 2.1 Deterioration in search effectiveness

We will now show that when an unemployed worker expects his/her search effectiveness  $\gamma$  is going to decrease (increase) at time  $t + 1$ , the likelihood of an active search increases (decreases). Note first that if  $\gamma_{t+1}$  is expected to go down then  $V_{U,t+1}$  must either remain constant or go down as well. To see this, consider the case when the optimal search intensity – before the decrease in  $\gamma_{t+1}$  is considered – was  $s_t = 0$ . After taking into account the decrease in

<sup>3</sup>the derivative of  $U_t$  with respect to search intensity  $s$  being obviously negative

$\gamma_{t+1}$  the optimal solution, and hence  $V_{U,t+1}$  must remain the same. Consider now the case when the optimal search intensity – before the decrease in  $\gamma_{t+1}$  is considered – was  $s_{t+1} = 1$ . After taking into account the decrease in  $\gamma_{t+1}$  the value of  $V_{U,t+1}$  has to go down, both if the new optimal solution is still  $s_{t+1} = 1$  (since the probability of finding a job goes down anyway) and if it becomes  $s_{t+1} = 0$  (this choice was already available before). But if  $V_{U,t+1}$  goes down the threshold  $\gamma_t^*$  goes up. Hence, the likelihood of an active search when the search effectiveness is still high increases. A symmetric argument holds when the unemployed worker expects his/her search effectiveness to increase.

### 3 Negative duration dependence in out-of-unemployment hazard rate

We now explicit a law of decay for search effectiveness as the unemployment spell increases. This amounts to introduce negative duration dependence in the hazard rate. As already stated, negative duration dependence might arise as a consequence of a loss of skills (at least with respect to those still at work), to a change in the structure and dimension of the network of people to whom the individual is connected, to stigma effects, to a general loss of capacity due to the prolonged reduction in income flows, etc. If unemployed workers are not able to anticipate this loss of employability we are back to the equilibrium framework where the expected value of being unemployed at time  $t + 1$  is the current value:  $E[V_{U,t+1}] = V_{U,t}$ . As a consequence, the threshold in eq. 5 does not change. In such a setting, this threshold would then be compared not with the *actual* search effectiveness at time  $t + 1$ , but with its *perceived* value. However, it would be curious to assume that individuals have (persistently) imperfect perception but perfect rationality. The whole model would look inappropriate and a more behavioristic specification should be preferred.

Here we consider the case where individuals have rational expectations about their search effectiveness, and correctly estimate that it decays at a constant rate  $\theta < 1$ :  $\gamma_{t+1} = \gamma_t(1 - \theta)$ . After some time  $T$  the search effectiveness will be so low that the unemployed individual will be better off stop searching:  $\gamma_t$  goes to 0, while  $\gamma_t^*$  converges to a positive value<sup>4</sup>, since  $U_{t>T} = U(0)$  and  $p_{t>T} = 0$ . Consequently,  $V_{U,t>T} = U(0)/(1 - R)$ .

To find  $T$ , we equal the value of  $V_{U,T}$  when  $s_T = 0$  and  $s_T = 1$ :

$$V_{U,T} = \frac{U(0)}{1 - R} = U(1) + \frac{U(0)R(1 - p_T)}{1 - R} + V_E R p_T \quad (6)$$

Hence,

$$(1 - \theta)^T = \frac{1 - R}{\gamma_0 R} \cdot \frac{U(0) - U(1)}{(1 - R)V_E - U(0)} \quad (7)$$

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<sup>4</sup>this value is given by  $\frac{1 - R}{R} \frac{U(0) - U(1)}{(1 - R)V_E - U(0)}$

Intuitively, the amount of time the individual undertakes active search increases in the initial skills  $\gamma_0$  and in the value attached to employment  $V_E$ , while decreases as the disutility from search  $U(1)$  and the depreciation rate  $\theta$  increase. Finally, note that if  $V_E < U(0)/(1 - R)$  the individual does not search at all.

## 4 Training courses

In this section, we deal with active labor market policies aimed at improving employability, such as training courses. Of course anyone can decide to undertake a training course, and bear the whole cost of it. But it could be in the social interest to encourage unemployed workers to attend retraining, if unemployment (which might lead to poverty) is considered a social problem *and* the private incentives are considered too low. So, we first turn to the problem of optimally choosing whether and when to undertake retraining from an individual point of view. We then explicit a social welfare function and investigate the problem from the point of view of a benevolent government. We will show that the solutions of the two problems generally differ, and discuss the implications. In particular, we will argue that training courses financed at least partly with public money should include among the eligibility criteria a minimum length of the unemployment spell. On the other hand we will also show that it could be optimal for an unemployed worker to turn down the free invitation to follow a training course, given the expectation that it will be possible (even at less advantageous conditions) to attend it in the future.

### 4.1 The problem of the individual

Suppose that following a training course that lasts for 1 period allows to recover the initial search effectiveness  $\gamma_0$ . The extension to the case when requalification is only partial or it takes longer is straightforward. By the way, note that the problem of choosing the optimal timing for retraining is identical to the problem of choosing the optimal timing of an investment in the replacement of a machinery. Let  $U_F$  be the utility deriving from following the course (this possibly includes the payment of a fee, the receipt of a scholarship, etc.). The optimal period  $T$  to undertake the course is characterized by the condition  $V_{U,t} = V_{U,T+1}$  (it is not convenient to wait any longer):

a) Program in  $t = T$ :

$$V_{U,T} = +U_F + RV_0 \equiv F \quad (8)$$

(b) Program in  $t = T + 1$ :

$$V_{U,T} = U_T + R[(1 - p_T)F + p_TV_E] \quad (9)$$

Hence:

$$p_T = \frac{(1 - R)F - U_T}{R(V_E - F)} \quad (10)$$

Now, there are 2 possibilities: (i)  $T$  happens when  $s = 1$  and the individual is still searching; (ii)  $T$  happens at a time when  $s = 0$  and the individual has already stopped searching. *Sub* (ii), the condition (a)=(b) becomes:

$$F^* = \frac{U(0)}{1 - R} \quad (11)$$

If  $F > F^*$  it is convenient to take the course in  $T$ ; otherwise it will never be convenient. By a continuity argument, if it is convenient to take the course in the first period when  $s = 0$  then it would still be convenient to enter the program in the period before, when the individual is still searching even if with a low probability of finding a job. Thus, it is possible to focus on case (i). The best moment for taking a training course is then identified by:

$$(1 - \theta)^T = \frac{(1 - R)F - U(1)}{\gamma_0 R(V_E - F)} \quad (12)$$

The optimal delay from the start of the unemployment spell increases in  $U(1)$ . As for what concerns the effect of  $F$ , note first that if the private cost of the training course (which lowers  $F$  and may reasonably bring it to a negative value) is too high, then  $(1 - R)F < U(1)$ . Assuming  $V_E > F$  (which is obviously true if  $F$  is negative), the numerator is negative while the denominator is positive and the individual will never choose to undertake retraining. The derivative of the RHS with respect to  $F$  is given by:

$$\frac{dRHS}{dF} = \frac{(1 - R)F - U(1)}{R(V_E - F)^2} + \frac{1 - R}{R(V_E - F)} \quad (13)$$

which is positive ( $T$  decreases) if the no-training condition does not hold –  $(1 - R)F > U(1)$  – while the “innocuous” condition  $V_E > F$  is still true. Thus, the optimal delay increases in the (private) cost of the program (which lowers  $U_F$  in eq. 8). The effects of  $V_E$  (hence of the wage  $w$ ) and  $\gamma_0$  on the other hand are ambiguous, since these two variables appear both at the denominator and at the numerator (as part of  $V_0$ , which is part of  $F$ ).

In conclusion, it is generally not convenient to undertake a training course *immediately* at the start of the unemployment spell, since the individual is still “fit enough” for the labor market. However, one could think that the firing probability is not entirely exogenous, and that those who loose a job have inadequate skills. If the training course allows to reach a level of employability  $\gamma_F$  higher than the one the unemployed worker has when fired,  $\gamma_0$ , the value of being unemployed after having completed the course is  $V_F > V_0$ ; hence the optimal delay decreases, and vanishes when  $\gamma_0 < \frac{(1 - R)F - U(1)}{R(V_E - F)}$ .

Note that according to our result of section 2.1 knowing that the search effectiveness is going to increase as a result of a training course implies that the likelihood of an active search *before* the course goes down. This however does not affect the argument above.

## 4.2 The problem of the society

Unemployment is generally regarded – at least in part – as a social problem. This might be due to a negative value attached to unemployment itself, or to the fact that unemployment lead to poverty and poverty is perceived as the ultimate social problem. In both cases there might be a general interest to lift people out of unemployment, or to increase the likelihood that they might get a job. Active labor market policies explicitly have this aim. So, if the cost of a training course is too high for the individual to bear it, it might be in the general interest to pay it (totally or partly).

In order to proceed with a formal analysis, suppose that the government attributes a cost  $D$  for each period of unemployment, irrespective of the length of the spell. The government can contribute at a cost of  $C$  to a training course that will increase his/her employability. Let  $E[u, t]$  be the expected residual length of the unemployment spell after  $t$  periods in unemployment. The average length of the unemployment spell is then  $E[u, 0] \equiv \bar{u}_0$ .<sup>5</sup> We assume that the government takes into consideration only the present spell, disregarding the possibility of future spells. Should this happen, it will pose a new problem to the government. Finally, we assume that the government discounts future events at their probability of occurrence.

Anticipating the timing of the training course has three effects: (i) it increases the likelihood that it will actually be provided (since if the unemployed worker finds a job the course is no longer necessary); (ii) it increases the probability the individual will find a job, conditional on being still unemployed at the time the course is finished; (iii) it decreases the relative advantage of taking the course, since the initial skills of the worker are higher. To find the optimal trade-off between these effects let's look again at the time when there is no more advantage in waiting (the negative contribution  $w_{i,t}$  of individual  $i$  to the social welfare function is minimal):

(a) Program in  $t = T$ :

$$w_{i,T} = -C - du_0 \quad (14)$$

where  $C$  is the public cost of the course and  $u_0$  the subsequent expected length of the unemployment spell.

(b) Program in  $t = T + 1$ :

$$w_{i,T+1} = -(1 - p_T)d - (1 - p_T)C - (1 - p_T)du_0 \quad (15)$$

where the individual has a probability  $p_T$  of finding a job in  $T$ , and takes the course only if still unemployed at time  $T + 1$ , coming back to the labor market at time  $T + 2$ .

Clearly, if the individual is not searching ( $p_T = 0$ ) it is convenient to anticipate the program: (a) > (b). Assuming  $T$  corresponds to a period in which the individual is still searching, we have:

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<sup>5</sup>When  $\theta = 0$ , i.e. with no deterioration in the employability, the average unemployment spell is  $u_0 = \frac{1}{\gamma_0}$ . In the general case  $\theta \neq 0$  the average unemployment spell is obviously higher, but it is still possible to show that it is finite (see the Appendix).

$$(1 - \theta)^T = \frac{d}{\gamma_0[C + d(u + 1)]} \quad (16)$$

Hence the optimal delay increases in the (public) cost of the program and in the average unemployment spell after having completed the program, while it decreases in the social cost that is attributed to unemployment (the derivative of the RHS with respect to  $d$  is positive). The effect of  $\gamma_0$  is ambiguous because when it increases it lowers the average length of an unemployment spell. However, if we suppose again that the training course leads to an employability  $\gamma_F$  independent of  $\gamma_0$ , then an increase in the skills the worker has when fired lead to postponing the course.

Ideally the government would fine-tune the share of the total cost of the training course paid by public money in order to let the solution to the individual problem outlined in the previous section coincide with the social optimum (given the burden on the public budget). This would also guarantee that training takes place at a moment when the individual is still actively searching, as we assumed. However, the solution of the individual problem depends on knowledge of the individual utility function. If it is already quite strong to assume that individuals are able to formalize a utility function for themselves, it is definitely too heroic to assume that it will ever be possible for the government to know these functions to any sufficient degree of accuracy. Moreover, since individuals differ with respect to the degree they value employment, unemployment, leisure, consumption etc., even if the government had this knowledge offering different offers to otherwise equal individuals would look arbitrary and be politically unfeasible.

So, the second best solution for the government would be to choose some average  $C$ , for instance by estimating the response of unemployed workers to the private cost of training. Then, depending on whether the government thinks that mandatory retraining affects the outcome of the training course<sup>6</sup>, either offer the opportunity – after a minimum of  $T$  periods of unemployment – of taking a subsidized training course or make it compulsory. Note that on average – given the subsidy  $C$  computed as outlined above – individuals will freely choose to undertake retraining exactly after  $T$  periods.

In conclusions we have stressed that – if employability decreases with the length of the unemployment spell – it is optimal to include a minimum length among the eligibility criteria for subsidized retraining, although it is optimal to offer retraining *before* unemployed workers become discouraged and stop searching for a job. This is not due to a morale effect, but to simple economic considerations. We have also shown that some unemployed workers might decide to postpone retraining even when it comes at no cost for them.

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<sup>6</sup>forced attendance might reduce active involvement, although this is debated at an empirical level (see references above)

## 5 Unemployment benefits

In this section we propose a unified treatment, within the decreasing employability framework of our model, of unemployment benefits and social assistance. We will show that it is generally optimal for the government to initially offer unemployment benefits, suspend them after some time in unemployment, and then offer unlimited income support schemes for those individuals that have stopped searching for a job. Intuitively this happens because at the beginning of the unemployment spell, when employability is still high, unemployed workers would search anyway, both with and without the benefit. The problems with unemployment benefits come when search effectiveness has already decreased, and thus the choice whether to search or not becomes more sensitive. As the duration of the unemployment spell further increases, it comes to a point when the individual would choose not to search even without the subsidy, thus removing the disincentives of income support schemes.

Let's begin with the trivial observation that unemployed workers would prefer to be granted the maximum amount of benefits, starting from the moment when they are fired, and possibly for the entire length of the unemployed spell. Because of the benefit the search intensity (or – in the linear case analyzed above – the likelihood of an active search) would go down. With decreasing employability the risk of being trapped in unemployment would then go up. However, since the unemployed could still undertake the same search effort that was optimal without the benefit, they would be inevitably better off.

On the other hand, if the goal is to fight unemployment the optimal choice from a social point of view would be to give no benefits at all.<sup>7</sup>

So, we assume that the government aims at reducing poverty, rather than unemployment. We also assume that (for the weakest class of workers) income in unemployment falls below the poverty line. The benefit covers exactly the gap<sup>8</sup>. The problem is that granting a benefit today on the one hand reduces the poverty rate today, while on the other hand increases the poverty rate tomorrow, since it possibly lowers job search.

Let's consider the decision of granting a benefit  $B$  at time  $t$ . As we have seen, without the subsidy poverty and unemployment coincide: the expected duration of a poverty spell is then equal to the expected duration of an unemployment spell,  $u_t$ . Thus we have:

(a) Without benefit in  $t$ :

$$w_{i,T} = -du_t \quad (17)$$

(b) With benefit in  $t$ :

$$w_{i,T} = -B - (1 - \tilde{p}_t)u_{t+1} \quad (18)$$

where  $\tilde{p}_t = \gamma_t \tilde{s}_t$  is the probability of finding a job given the optimal level of search intensity with the benefit,  $\tilde{s}$ . We must then distinguish three cases: (i)

<sup>7</sup>We do not consider the case when the subsidy allows for a better search, hence leading to a better quality of the match, which finally results in a lower separation probability and consequently a lower unemployment rate.

<sup>8</sup>rather than an assumption this is a consequence of the specific goal of the government

$s_t = \tilde{s}_t = 1$ , active search both with and without the benefit; (ii)  $s_t = 1; \tilde{s}_t = 0$ , active search only without the benefit; (iii)  $s_t = \tilde{s}_t = 0$ , no search both with and without the benefit.

Let's start with the latter. *Sub* (iii) the expected duration of the poverty spell, given the individual is poor (i.e. unemployed) in  $t$  is  $l_t$ , where  $l$  is the residual (working) life. It is then optimal to grant the subsidy whenever  $-dl_t < -B - d(l_t - 1)$ , or:

$$B < d \tag{19}$$

that is the cost of the subsidy is smaller than the value attached to lift the individual out of poverty.

*Sub* (ii) the condition becomes  $-du_t < -B - du_{t+1}$ , or:

$$B < d(u_t - u_{t+1}) \tag{20}$$

which covers both the cases when  $t + 1$  is covered by the unemployed insurance and when it is not.<sup>9</sup> Now, with active search in  $t$  (and irrespective of what happens in  $t + 1$ )  $u_t$  and  $u_{t+1}$  are linked by the relation

$$u_t = \gamma_t + (1 - \gamma_t)(u_{t+1} + 1) = 1 + (1 - \gamma_t)u_{t+1} \tag{21}$$

Being unemployed at time  $t$  leads with a probability of  $\gamma_t$  to a duration of 1 and with a probability of  $1 - \gamma_t$  to a duration of  $u_{t+1} + 1$ . Note that the expected residual duration of the unemployment spell is increasing (i.e.  $u_{t+1} > u_t$ ) if  $u_t > \frac{1}{\gamma_t}$ . The condition finally becomes:

$$B < d(1 - \gamma_t u_{t+1}) = d\Delta \tag{22}$$

which is more restrictive than in case (iii) since  $\Delta < 1$ .

Finally, *Sub* (iii) granting the subsidy is optimal given  $-du_t < -B - (1 - \gamma_t)du_{t+1}$  and substituting eq. 21 we obtain again condition 19:  $B < d$ .

Now, assuming  $B < d$  the problem is that the government cannot discriminate between these three cases. Moreover, there is a strong incentive for unemployed workers in case (ii) to stop searching and claim to be in case (iii), or to claim they will not stop searching with the benefit to be included in case (i). To proceed further in the analysis we must then specify a particular functional form for the individual utility function.

Let's assume a Cobb-Douglas form  $U = CL$  (which meet the constraint of linearity in  $s$ ), with  $L = 1 - s$ . This implies  $U(1) = 0$  and  $U(0) = C$ . Consumption is at a minimal level  $C = \bar{C}$  without the benefit, while it raises to  $C = \bar{C} + B$  with the benefit. In order to compute the moment when the individual stops searching we keep on our initial assumption that the individual has infinite life and equal the value of  $V_{U,T}$  in eq. 3 when  $s_T = 0$  and  $s_T = 1$ , as in eq. 27. We consider that at time  $T$  the individual has  $n$  remaining periods of

<sup>9</sup>If  $t + 1$  is covered by the unemployed insurance, or if it is not but the individual stops searching anyway ( $s_{t+1} = 0$ ), we simply have  $u_{t+1} = l_t - 1$ .

unemployment benefits, and will get an equal amount as social assistance from time  $T + m$  on. This assumption will be confirmed by the analysis below.

$$V_{U,T} = \frac{\bar{C} + B}{1 - R} - \frac{B(R^{n+1} - R^m)}{1 - R} = R(1 - p_T) \frac{\bar{C} + B(1 - R^n + R^{m-1})}{1 - R} + V_E R p_T \quad (23)$$

Hence:

$$(1 - \theta)^T = \frac{(1 - R)(\bar{C} + B)}{\gamma_0 R [(1 - R)V_E - \bar{C} - B(1 - R^n + R^{m-1})]} \quad (24)$$

Note that if there are no benefits ( $B = 0$ ) the moment when the individual stops searching comes later (the numerator goes down while the denominator goes up).

Let's assume for simplicity that  $\delta = 0$  (no separation, once attached) and  $E = w$  (the utility deriving from employment is given by the salary). Hence,  $V_E = w/(1 - R)$ .

The edge between case (i) and case (ii) is then at  $t = T_1$ , defined as the moment when the individual stops searching with the benefits, and the edge between case (ii) and case (iii) is at  $t = T_2$ , defined as the moment when the individual stops searching when there are no benefits:

$$\begin{aligned} T_1 &= \frac{\log(1 - R) + \log(\bar{C} + B) - \log(\gamma_0 R) - \log(w - \bar{C} - B(1 - R^n + R^{m-1}))}{\log(1 - \theta)} \\ T_2 &= \frac{\log(1 - R) + \log(\bar{C}) - \log(\gamma_0 R) - \log(w - \bar{C})}{\log(1 - \theta)} \end{aligned} \quad (25)$$

Note that if  $w$  is high enough an increase in the amount of the benefit  $B$  shortens the period covered by the unemployment insurance  $T_1$ <sup>10</sup>; however, it does not affect the timing of social assistance.

Given  $B < d$  it is optimal to give the benefit in case (i) and (iii). But what about case (ii)? Looking back at eq. 22, we observe that as time in unemployment goes by  $\gamma_t$  decreases but  $u_{t+1}$  might well increase, if the condition  $u_t > 1/\gamma_t$  holds. Hence, the value of  $\Delta$  is *a priori* indeterminate. In order to go further we have simulated eq. 21. It turns out that the crucial parameter is  $\theta$ , the employability depreciation rate. As an example, suppose to normalize the wage  $w$  to 10. Suppose the minimum consumption  $\bar{C}$  is 30% of the wage, i.e. 3, the poverty line is at 50% of the wage and thus the benefit  $B$  is equal to 2. Consider a discount rate  $R = .95$  and an initial probability of finding a job  $\gamma_0 = .5$ . Finally, suppose that the working lifetime of an individual is bounded

<sup>10</sup>the derivative of  $T_1$  with respect to  $B$  being negative:

$$\frac{dT_1}{dB} = \frac{w - \bar{C}(R^n - R^{m-1})}{(\bar{C} + B)(w - \bar{C} - B(1 - R^n + R^{m-1}))\log(1 - \theta)} \quad (26)$$

at  $l_0 = 500$  (think of 500 months, i.e. about 40 years).<sup>11</sup> Figure 1 reports the expected unemployment spell  $u_t$  and the value of  $\Delta$  for different values of  $\theta$ . When time exceeds  $T_2$ , i.e. when the individual definitely stops searching the expected residual length of the unemployment spell is equal to the residual lifetime, i.e.  $500 - l_t$ . Before  $T_2$  it is computed using eq. 21. The expected residual length of the unemployment spell  $u$  turns out to be monotonically increasing up to  $T_2$ , when the individual definitely stops searching and the residual length coincides with the residual working lifetime. The highlighted areas correspond to the range  $[T_1, T_2]$ : in this range the values of  $\Delta$  are always negative, hence the benefits are not granted.

Only when  $\theta$  is very low, as in figure 2, the expected residual length of the unemployment spell starts to decrease before  $T_2$ , and  $\Delta$  is positive in the final part of the range  $[T_1, T_2]$ : in this case it would be optimal to start the social assistance before  $T_2$ .

If we rule out such low values of the depreciation rate it is optimal to suspend any kind of assistance between  $[T_1, T_2]$ . Thus  $n$ , the remaining periods of unemployment benefits after  $T_1$ , becomes equal to 0, and  $m$ , the number of period after  $T_1$  when social assistance starts, becomes equal to  $T_2 - T_1$ . Hence the expression of  $T_1$  in eq. 25 becomes:

$$T_1 = \frac{\log(1 - R) + \log(\bar{C} + B) - \log(\gamma_0 R) - \log(w - \bar{C} - BR^{T_2-1})}{\log(1 - \theta)} \quad (27)$$

## 6 Conclusions

In this paper we have proposed a non-equilibrium model in order to explain the search behavior of unemployed workers. This modeling strategy, framed in a rational choice paradigm, allows us to investigate the effects of negative duration dependence in the out-of-unemployment hazard rate, accounting for a decrease in employability as the unemployment spell lengthens. We have showed that individuals react to an expected reduction in their search effectiveness by increasing their search efforts. We then analyzed two policies aimed at reducing poverty. One is an active labor market policy consisting in training courses to improve the employability of unemployed workers. We showed that it is optimal for the government to include among the eligibility criteria for subsidized training a minimum length of the unemployment spell. This spell however must be short enough for the unemployed workers not to become discouraged and stop searching. The second is an income support scheme. We showed that for a broad range of the parameters the optimal scheme takes the form of unemployment benefits granted for a limited amount of time starting from the beginning of the unemployment spell, coupled with social assistance for long-term unemployed with very limited residual employability.

Considering the possibility of both income support schemes and retraining, a typical government intervention schedule will thus look like in figure 3. Training

<sup>11</sup>All the results are robust to variations in these parameters.

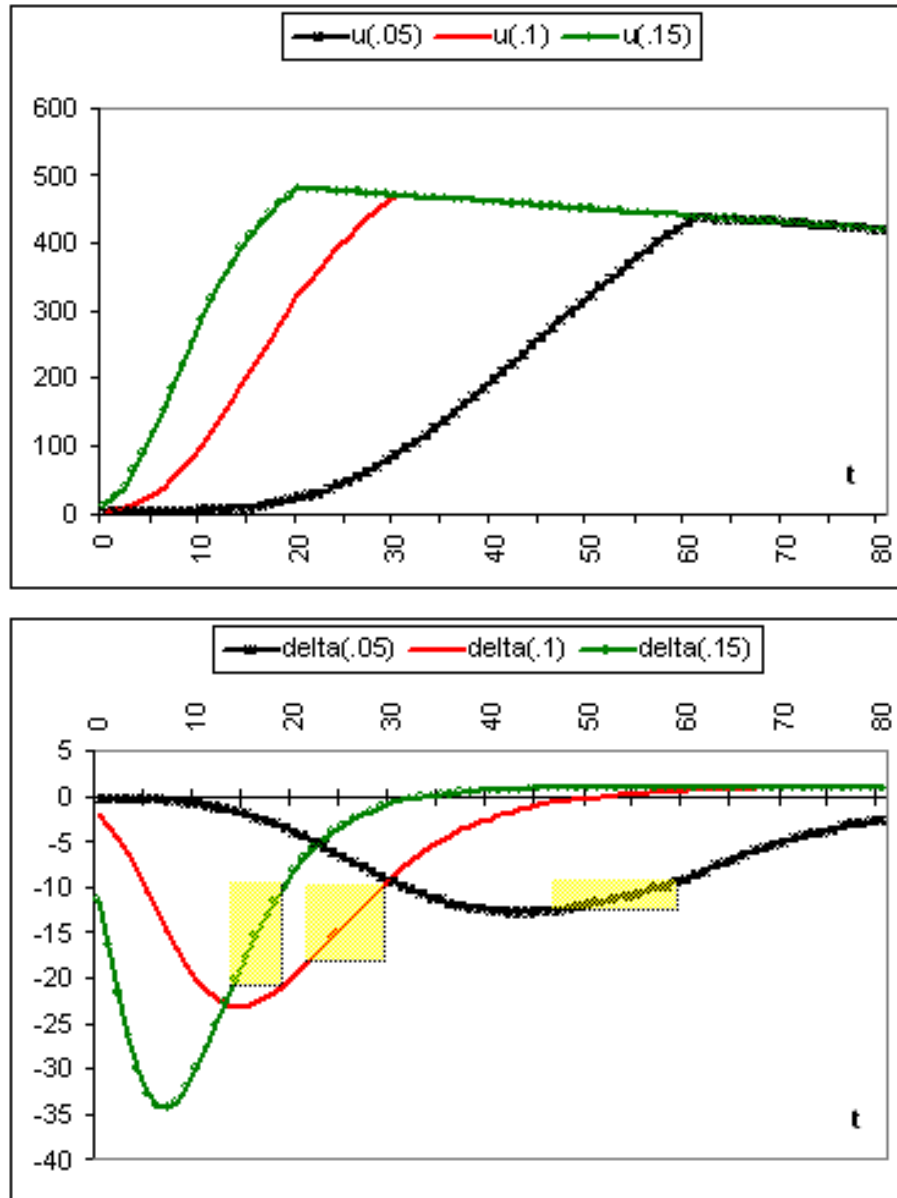


Figure 1: Average length of unemployment spell  $u_t$ (top) and value of  $\Delta$  in eq. 22 (bottom), different values of  $\theta$  (in parenthesis). When  $\Delta < 0$  the condition for granting the benefit cannot hold. The highlighted areas correspond to the range  $[T_1, T_2]$ .

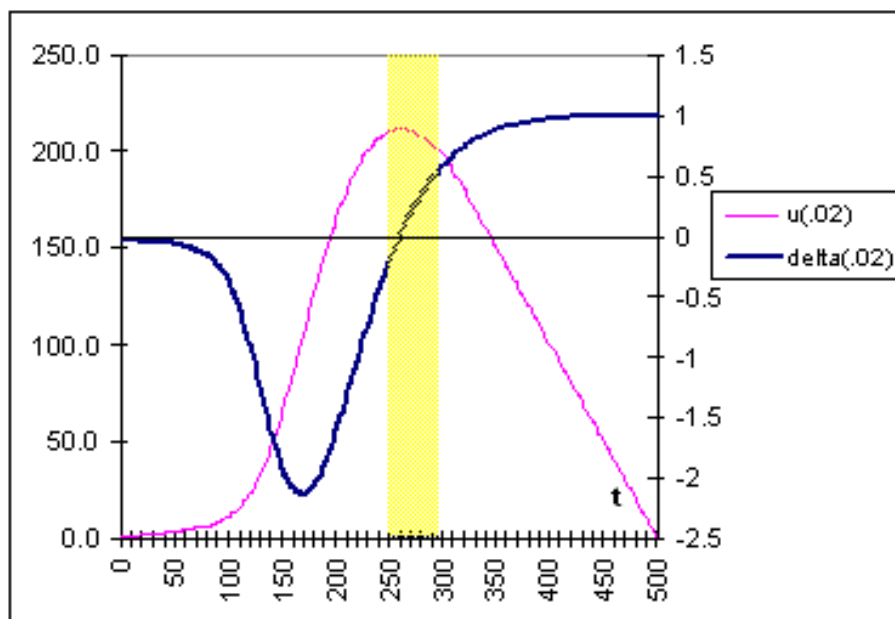


Figure 2: Average length of unemployment spell  $u_t(\text{top})$  and value of  $\Delta$  in eq. 22 (bottom),  $\theta = .02$ . When  $\Delta < 0$  the condition for granting the benefit cannot hold. The highlighted areas correspond to the range  $[T_1, T_2]$ .

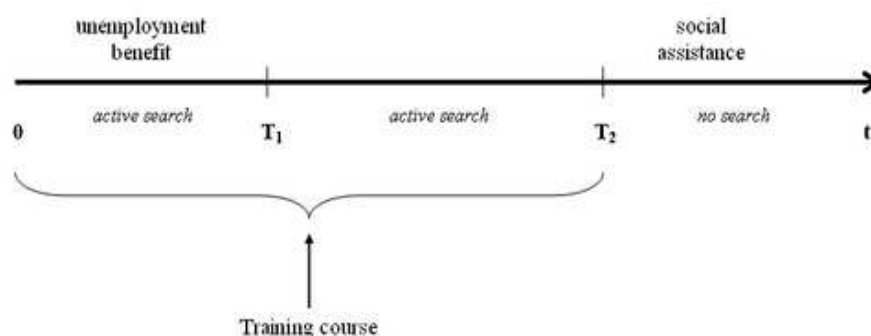


Figure 3: Income support scheme

courses can be offered both during the period when unemployed workers are granted unemployment benefits and after, although they should not be offered at a time when the individuals have already entered a minimum income scheme.

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## A Unemployment duration

### A.1 No decay in the hazard( $\theta = 0$ )

When search effectiveness remains constant over time, i.e.  $\gamma_t = \gamma \forall t$ , the average duration of one unemployment spell is given by:

$$u_0 = \gamma \sum_{n=0}^{\infty} (n+1)(1-\gamma)^n \quad (28)$$

The series converges to  $\left(\frac{1}{\gamma}\right)^2$ ; hence  $u_0 = \frac{1}{\gamma}$ .

### A.2 Linear decay in the hazard

If, as assumed in the model, search effectiveness linearly decreases over time such as  $\gamma_{t+1} = \gamma_t(1-\theta)$ , the average duration of one unemployment spell is given by:

$$u_0 = \gamma_0 \sum_{n=0}^{\infty} (n+1)(1-\theta)^n \prod_{j=0}^{n-1} (1-\gamma_0(1-\theta)^j) \quad (29)$$

The series converges to  $\left(\frac{1}{\theta}\right)^2$ ; hence  $\gamma_0 \sum_{n=0}^{\infty} (n+1)(1-\theta)^n$  adds up to  $\gamma_0 \left(\frac{1}{\theta}\right)^2$ . Since  $\prod_{j=0}^{n-1} (1 - \gamma_0(1-\theta)^j) < 1$  for any value of  $\gamma_0$  and  $\theta$  in  $(0,1)$ , the average duration  $u_0$  is finite.